Construction of Curvelet Transform as an Extension of Wavelet Transform

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Abstract. Curvelet Transform typically offers significantly superior performance in image analysis, multi-resolution and multidirectional representation as compared to Wavelet Transform. This paper exploites strong relationship between Wavelet Transform and Curvelet Transform. Also we use mother Wavelet to construct Curvelet Transform as an extension of Wavelet Transform which has broad implications, particularly for the field of signal and image processing.

INTRODUCTION

Signal and image processing, biological and computer vision, scientific computing, and optical data analysis have all extensively employed wavelet-based multi-resolution techniques [1]. Olshausen and Field's work in Nature [2], The similarities between multi-scale image processing and vision have been discussed by researchers studying biological vision. Wavelet transform as an extension of fractional Fourier transform is useful to solve ordinary differential equations and partial differential equations like heat equation, schrodinger's equation [3]. However, a limitation of wavelets is their direction selectivity, which is a crucial response characteristic for simple cells and neurons at different points along the visual pathway. Recent years have seen significant progress with the development of directional wavelets, surpassing several earlier attempts. Modeling and analyzing traffic networks is just one of the many useful applications of network analysis [4].

Candes and Donoho proposed an anisotropic geometric curvelet transform in1999. Later, Candes and Donoho proposed a much simpler second generation curvelet transform based on a frequency partition technique [5]. In image processing, digital images are represented as two-dimensional matrices. To obtain clear features in images, one crucial task is to modify the values of these matrices [6]. Currently, the second-generation curvelet transform finds extensive application across various fields, including fluid mechanics, seismic data exploration, signal and image processing, and solution of partial differential equations arising in nonlinear physical phenomena. Curvelet functions represent functions which have discontinuities along straight lines where wavelet functions are not properly worked [7]. The curvelet transform is a recent development used in signal and image processing applications. This study systematically transitions from classical wavelets to curvelets. So we are finding an extended curvelet transform via wavelet transform.

THE CLASSICAL WAVELET TRANSFORM

Wavelets are mathematical functions that perform scale or resolution-based data analysis. When analysing a signal across different windows or resolutions, wavelets prove useful. In practice, a wavelet transform involves convolving the signal with a set of functions derived from shifts and dilations of a fundamental wavelet. In exact words and notation the classical wavelet transform, also known as the Continuous Wavelet transform (CWT), is a decomposition of a function, f(x), with respect to a basic wavelet, $\psi(x)$, represented by the convolution of a function with a scaled