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Self-focusing/defocusing of skew-cosh-Gaussian laser beam for collisional plasma

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Abstract

The inverse relationship between the linear increase in skewness parameter s and the domain's width of the order of skewness n plays a vital role in both critical beam radius and propagation dynamics of skew-cosh-Gaussian (skew-chG) laser beams. The interplay between the skewness parameter s and the order of skewness n is explored analytically and graphically in the current study to unveil the complexity of the propagation dynamics of the skew-chG laser beam. Naturally, the intensity's complexity considerably affects the dielectric constant of the medium. Basically, the nonlinearity in the dielectric function of collisional plasma is attributed to the non-uniform heating of energy carriers along the wavefront of the laser beam, which becomes important and is used in the current study. By following Akhmanov's parabolic wave equation approach under Wentzel–Kramers–Brillouin and paraxial approximations, the nonlinear differential equations are set up for the beam width parameters f_1 and f_2 and solved numerically. The present work analytically investigates the domains of the order n of skew-chG laser beams for a given set of skewness parameter s to investigate their effects on the self-focusing and defocusing of skew-chG laser beams. The critical curve also gets affected significantly due to the choice of domains for n. Finally, the numerical results are presented in the form of graphs and discussed in this study.

Keywords: skew-cosh-Gaussian, skewness parameters, self-focusing/defocusing, collisional plasma

1. Introduction

Nonlinear refraction enables the laser to self-focus and self-guide across large distances in the plasma, thereby accounting for the diffraction divergence. A great number of experimental and theoretical investigations have been conducted on the self-focusing of laser beams [1, 2]. Self-focusing and defocusing of laser beams in the nonlinear media have been studied by Akhmanov *et al* [1]; moreover, Sodha *et al* [2] conducted its pedagogical straightforward extension to plasmas. Due to significant advances in laser technology, lasers have become the most powerful coherent source of radiation. The field of high-power laser plasma has seen phenomenal growth after the increase in laser power. The laser-plasma interactions have miscellaneous applications in higher harmonic generation [3], laser acceleration of electrons and ions [4], laser ablation of materials [5], inertial confinement fusion [6], laser coupling to graphene plasmonics [7], x-ray lasers [8], stimulated Raman scattering [9, 10], and self-phase modulation [11], etc.

Basically, the collisional plasma dynamics is dominated by local collisional forces rather than collective actions in it.

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An important factor in the field-dependent dielectric function in the collisional plasma is the non-uniform rearrangement of carriers caused by the heterogeneous heating of carriers because of transverse changes in the electric field along its wavefront. A laser beam propagating with an inhomogeneous intensity distribution in the plasma heats electrons, leading to a temperature gradient. The non-uniform heating of the carrier mechanism is observed to be effective rather than the ponderomotive force mechanism in a steady state. As a consequence of the self-induced inhomogeneity in the dielectric function of the plasma, the effective dielectric function modifies remarkably, and the self-focusing and defocusing of the beams are produced [12–14]. Recently, some laser beam profiles have been explored for the studies of self-focusing and defocusing, such as Gaussian beams [15–17], cosh-Gaussian (ChG) beams [18–21], Hermite-ChG beams [22–25], among others, in collisional plasmas. These techniques have attracted the attention of many research scholars. In contrast, some of the studies have been conducted on the self-focusing of quadruple Gaussian [26], q-Gaussian [27–30], Laguerre Gaussian [31, 32], Bessel–Gaussian [33], skew-chG [34, 35], and Airy–Gaussian [36].

Recently, Singh *et al* [37] studied the laser-plasma interactions where a skew-chG laser beam can create Wakefield. They have reported that for the generation of terahertz (THz) radiation, the order n and skewness parameters s of skew-chG laser beams are found to be very efficient in weakly ionized and collisional plasmas. Malik [38] has studied a generalized treatment of the skew-chG laser for bifocal THz radiation and reported that the propagation of skew-chG laser beams in collisional plasma is efficient for achieving unifocal bi-focal or unifocal THz radiation by the appropriate selection of skewness parameter s and order n of the skew-chG laser beam. In the present work, we have explored the electric field profile of a skew-chG laser beam under the symmetry condition in two transverse directions. For the given skewness parameter $s = 0.5, 1.0, 1.5, 2.0$ [38], the domains of order n are investigated analytically as they play a key role in determining the intensity profile of the laser beams. It has also been emphasized that the skewness parameter s and order n of the skew-chG laser beams play a crucial role in both the critical beam radius and the propagation dynamics of the skew-chG laser beams. Moreover, the analytical investigation in the present study focuses on the in-depth exploration of order n and skewness parameter s of skew-chG laser beams. More precisely, the present work analytically investigates the domains of the order n of a skew-chG laser beam for a given set of skewness parameter s to examine its effects on the propagation dynamics of skew-chG laser beams. By following Akhmanov's parabolic wave equation approach under Wentzel–Kramers–Brillouin (WKB) and paraxial approximations. Eventually, the beam-width parameter (BWP) f_1 and f_2 differential equations are set up and solved numerically and the obtained numerical results are presented in the form of graphs and discussed at the end.

2. Theoretical formulation

The propagation of skew-chG laser beams with angular frequency ω is considered in homogeneous plasma along the z axis. The initial electric field profile for skew-chG laser beams in Cartesian co-ordinate system can be written as:

$$E(x,y,z) = E_0 \cosh^n\left(\frac{s_x\, x}{r_0}\right) e^{-\left(\frac{x^2}{r_0^2}\right)} \cosh^m\left(\frac{s_y\, y}{r_0}\right) e^{-\left(\frac{y^2}{r_0^2}\right)} \tag{1}$$

where s_x and s_y are skewness parameters along the x- and y-directions, respectively, n and m are order of skewness, r_0 is the initial beam radius, E_0 is the maximum amplitude at the central position ($x = y = z = 0$). For convenience, we can express equation (1) as follows:

$$E(x,y,z) = \frac{E_0}{2^{(n+m)}} \exp\left(\frac{n^2 {s_x}^2}{4}\right) \exp\left(\frac{m^2 {s_y}^2}{4}\right) \times \left\{ \begin{array}{l} \left[e^{-\left(\frac{x}{\sqrt{n}\, r_0} + \frac{\sqrt{n}\, s_x}{2}\right)^2} + e^{-\left(\frac{x}{\sqrt{n}\, r_0} - \frac{\sqrt{n}\, s_x}{2}\right)^2} \right]^n \\ \left[e^{-\left(\frac{y}{\sqrt{m}\, r_0} + \frac{\sqrt{m}\, s_y}{2}\right)^2} + e^{-\left(\frac{y}{\sqrt{m}\, r_0} - \frac{\sqrt{m}\, s_y}{2}\right)^2} \right]^m \end{array} \right\} \tag{2}$$

The propagation of skew-chG laser beams in the homogeneous plasma is described by a dielectric function which can generally be written as [2]:

$$\varepsilon = \varepsilon_0 + \phi\ (EE^*), \tag{3}$$

where ε_0 and $\phi\ (EE^*)$ represent the linear and nonlinear parts of dielectric function ε, and $\varepsilon_0 = 1 - (\omega_p^2/\omega^2)$, where ω_p is the plasma electron frequency in the absence of the beam and $\omega_p = \sqrt{4\pi\ n_e\ e^2/m}$, where and e are the density of plasma electrons in the absence of the laser beam, and charge on electrons, respectively. The term $\phi\ (EE^*)$ represents a field-dependent dielectric function for collisional plasma [14]:

$$\phi\,(EE^*) = \frac{\omega_p^2}{\omega^2}\left[\frac{\alpha\, EE^*}{2 + \alpha\, EE^*}\right], \tag{4}$$

with

$$\alpha = \left(\frac{e^2 M}{6\, k_B T_0 \omega^2 m^2}\right),$$

where k_B, M, n and T_0 are the Boltzmann constant, the ion mass, the rest mass of the electron, and the plasma temperature, respectively.

The wave equation governing the electric field of the laser beam, having dielectric function ε of the plasma given by equation (3), can be expressed as follows:

$$\nabla^2 E - \frac{\varepsilon}{c^2}\frac{\partial^2 E}{\partial t^2} = 0. \tag{5}$$

In WKB approximation, the solution of equation (5) can be written as follows:

$$E = A(x,y,z)\exp[i(\omega t - kz)]. \quad (6)$$

Substituting ε and E from equations (3) and (6) in equation (5) and solving, one can obtain:

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\omega^2}{c^2}\phi(EE^*)A = 2ik\frac{\partial A}{\partial z}. \quad (7)$$

To solve parabolic wave equation (7), we now express $A(x,y,z)$ as follows:

$$A(x,y,z) = A_0(x,y,z)\exp[-ikS(x,y,z)] \quad (8)$$

where $A_0(x,y,z)$ and $S(x,y,z)$ are the real functions of x, y and z. Substituting for A from equation (8) in equation (7) and equating real and imaginary parts, we get:

$$2\left(\frac{\partial S}{\partial z}\right) + \left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial y}\right)^2 = \frac{1}{k^2 A_0}\left(\frac{\partial^2 A_0}{\partial x^2} + \frac{\partial^2 A_0}{\partial y^2}\right) + \frac{\omega_p^2}{\omega^2 \varepsilon_0}\left[\frac{\alpha EE^*}{2+\alpha EE^*}\right] \quad (9)$$

and

$$\frac{\partial A_0^2}{\partial z} + \left(\frac{\partial S}{\partial x}\right)\left(\frac{\partial A_0^2}{\partial x}\right) + \left(\frac{\partial S}{\partial y}\right)\left(\frac{\partial A_0^2}{\partial y}\right) + \left(\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2}\right)A_0^2 = 0. \quad (10)$$

Following the approach given by Akhmanov *et al* [1] and Sodha *et al* [2], the solutions of the equations (9) and (10) are expressed as follows:

$$S = \beta_1(z)\frac{x^2}{2} + \beta_2(z)\frac{y^2}{2} + \phi(z) \quad (11)$$

and

$$A_0^2(x,y,z) = \frac{E_0^2}{4^{(n+m)} f_1(z) f_2(z)} \exp\left(\frac{n^2 S_x{}^2}{2}\right)\exp\left(\frac{m^2 S_y{}^2}{2}\right) \times \left\{ \begin{pmatrix} \exp\left[-2\left(\frac{x}{\sqrt{n}f_1(z)r_0} - \frac{\sqrt{n}\,s_x}{2}\right)^2\right] \\ +\exp\left[-2\left(\frac{x}{\sqrt{n}f_1(z)r_0} + \frac{\sqrt{n}\,s_x}{2}\right)^2\right] \\ +2\exp\left[-\left(\frac{2x^2}{n\,r_0^2 f_1(z)^2} + \frac{n\,s_x^2}{2}\right)\right] \end{pmatrix}^n \begin{pmatrix} \exp\left[-2\left(\frac{y}{\sqrt{m}f_2(z)r_0} - \frac{\sqrt{m}\,s_y}{2}\right)^2\right] \\ +\exp\left[-2\left(\frac{x}{\sqrt{m}f_2(z)r_0} + \frac{\sqrt{m}\,s_y}{2}\right)^2\right] \\ +2\exp\left[-\left(\frac{2y^2}{m\,r_0^2 f_2(z)^2} + \frac{m\,s_y^2}{2}\right)\right] \end{pmatrix}^m \right\} \quad (12)$$

It should be noted that $\beta_1(z) = (1/f_1)(\partial f_1(z)/\partial z)$, $\beta_2(z) = (1/f_2)(\partial f_2(z)/\partial z)$ where $\beta_1(z)$ and $\beta_2(z)$ are the inverse of the radius of curvature of the beam along the x and y directions respectively, $\phi(z)$ is the axial phase, $f_1(z)$ and $f_2(z)$ are dimensionless BWP along the x and y directions of the beam, respectively.

Substituting equations (11) and (12) in equation (9), we have obtained differential equations for BWP f_1 and f_2 of skew-chG laser beams as follows:

$$\frac{d^2 f_1}{d\xi^2} = \frac{n^2 s_x^4 - n\,s_x^4 - 4\,n\,s_x^2 + 4}{f_1{}^3} - \frac{2\,(2 - n s_x{}^2)\,\alpha E_0^2\, f_2\, r_0^2\,\omega_p^2}{(\alpha E_0^2 + 2 f_1 f_2)^2\, c^2} \quad (13)$$

and

$$\frac{d^2 f_2}{d\xi^2} = \frac{n^2\, s_y^4 - n\,s_y^4 - 4\,n\,s_y^2 + 4}{f_2{}^3} - \frac{2\,(2 - n s_y{}^2)\,\alpha E_0^2\, f_1\, r_0^2\,\omega_p^2}{(\alpha E_0^2 + 2 f_1 f_2)^2\, c^2} \quad (14)$$

where $\xi = z/R_d$ is the dimensionless distance of propagation and $R_d = k\,r_0{}^2$ is known as Rayleigh length with $k = (\omega/c)\sqrt{\varepsilon_0}$ is the wave number. Now, we have made analytical investigation by considering the same skewness parameters $s_x = s_y = s$ and the order of skewness is also the same, that is $n = m$ of the skew-chG laser beam along x and y directions, respectively.

Equations (13) and (14) are nonlinear, coupled second-order differential equations, which show the variation of the BWP f_1 and f_2 with respect to the dimensionless distance of propagation ξ. The first term on the right-hand side of these equations leads to the diffraction divergence, which is responsible for defocusing and the second term leads to convergence resulting from the collisional nonlinearity, which is responsible for self-focusing. When the self-focusing and diffraction of a laser beam are perfectly balanced, the beam becomes self-trapped.

3. Results and discussion

By imposing initial conditions on equations (13) and (14), $f_1(\xi=0) = f_2(\xi=0) = 1$ and $d^2 f_1/d\xi^2 = d^2 f_2/d\xi^2 = 0$ under symmetry considerations, such as $s_x = s_y = s$ and $n = m$ along x and y directions, we obtain an expression for the dimensionless critical initial beam radius $\rho_0 = \omega_p r_0 / c$ in terms of the critical intensity parameter $p = \alpha E_0^2$ as follows:

$$\rho_0 = \sqrt{\frac{(n^2 s^4 - n s^4 - 4\,n\,s^2 + 4)\,(p+2)^2}{2\,p\,(2 - n s^2)}}. \quad (15)$$

Now equation (15) can also be written as:

$$\frac{1}{\rho_0{}^2} = \frac{2\,p\,(2 - n s^2)}{(n^2 s^4 - n s^4 - 4\,n\,s^2 + 4)\,(p+2)^2}. \quad (16)$$

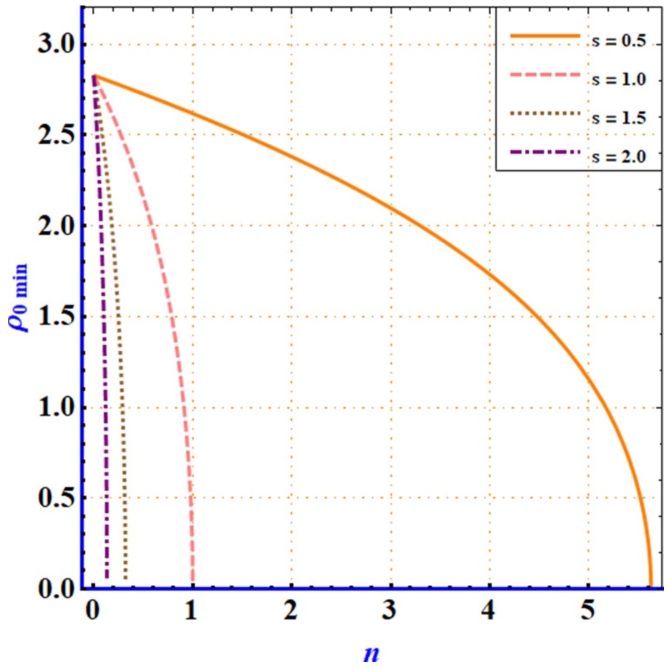

Figure 1. Domains of order n of skew-chG laser beam with different values of skewness parameter s.

For the maximum value of right hand side, we can rewrite equation (16) as:

$$\frac{d}{dp}\left[\frac{2\,p\,\left(2-ns^2\right)}{\left(n^2s^4-ns^4-4\,n\,s^2+4\right)\,\left(p+2\right)^2}\right]=0. \qquad (17)$$

By solving equation (17), we obtain:

$$p=2. \qquad (18)$$

Substituting the value of p into equation (15), we obtain:

$$\rho_{0\min}=\frac{4\,\left(n^2s^4-ns^4-4\,n\,s^2+4\right)}{\left(2-ns^2\right)}. \qquad (19)$$

Equation (19) can be used for the further analytical and graphical explorations. The expression of $\rho_{0\min}$ is purely dependent on skewness parameter s and order of skewness n. Figure 1 illustrates the variation of $\rho_{0\min}$ against the order of skewness n for a given set of values of skewness parameters s (0.5, 1.0, 1.5, and 2.0) [38] and highlights the domains of order of skewness n. The subsequent analytical investigation using equation (19) under the condition $\rho_{0\min}>0$ results in four different domains corresponding to each value of skewness parameter s. The reconfirmation of the domains of order of skewness n is quite evident from figure 1.

It is a well known fact that the critical curve investigation results in a clear discrimination of above critical curve (supercritical region) and below critical curve (subcritical region). These two regions are always separated by critical curve. Thus, using equation (15), figure 2 gives four sets of plots of critical curves where each set corresponds to a given value of s. In those sets, in general, it is observed that with increase in the value of n, the critical curve shifts downward and saturates at its minimum value. For a point $(\rho_0,\,p_0)$ in the supercritical region, self-focusing is observed. For a point $(\rho_0,\,p_0)$ in the subcritical region, defocusing is observed. Naturally, however, for any value of $(\rho_0,\,p_0)$ on the critical curve, self-trapping of the skew-chG laser beam is observed, which is obviously self-explanatory. Finally, the coupled differential equations (13) and (14) are solved numerically for the values of $\rho_0 = 3.0000$ (supercritical region), $\rho_0 = 0.5550$ (subcritical region) and $p_0 = 2.0000$, which are consistent with all the plots of critical curves in figure 2.

In figure 3, we have shown explicitly the variation of BWP f_1 and f_2 as a function of dimensionless propagation distance ξ for various domains of the order n of skew-chG laser beam (table 1) along with $\rho_0 = 3.0000$ and 0.5550 and $p_0 = 2.0000$. The variations in the BWP f_1 and f_2 with respect to the domains of n are clearly evident in figure 3. We have selected two distinct values of $\rho_0 = 3.0000$ and 0.5550 with $p_0 = 2.0000$, which are present in the supercritical region and in the subcritical region, respectively. The self-focusing

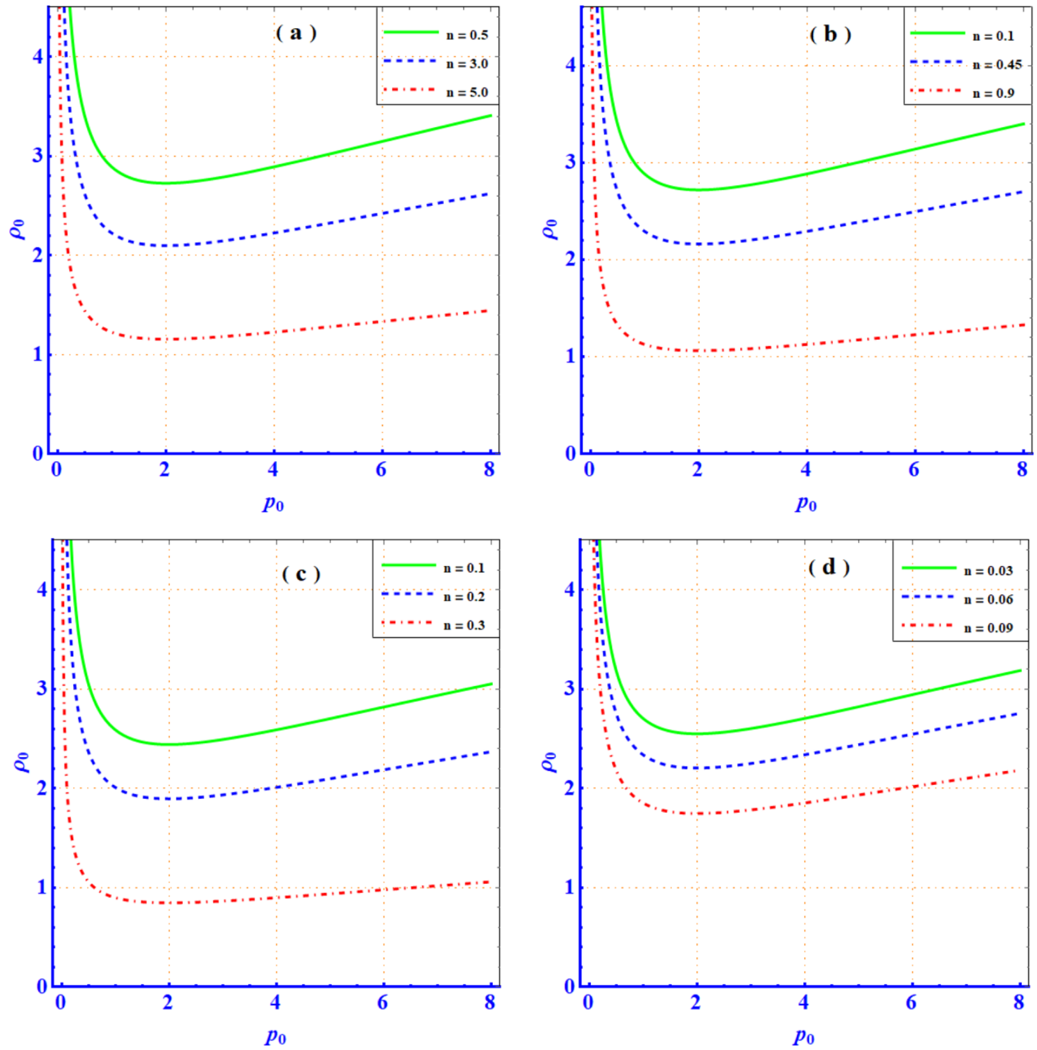

Figure 2. Critical curves for different values of skewness parameters s and same order of skewness (symmetry) $n = m$ (a) $s = 0.5$, (b) $s = 1.0$, (c) $s = 1.5$, and (d) $s = 2.0$.

($\rho_0 = 3.0000$ and $p_0 = 2.0000$) and the defocusing of the laser beam ($\rho_0 = 0.5550$ with $p_0 = 2.0000$) with respect to the domains are (table 1) are observed. For any value of (ρ_0, p_0) on the critical curve, the self-trapping of the skew-chG laser beam is obviously self-explanatory. In addition, for a given skewness parameter s with respect to domains of the order of the skew-chG laser beam n the rate of defocusing decreases. Figure 3 represents that when the domains of n increase along with s, the self-focusing length decreases, resulting in oscillatory self-focusing.

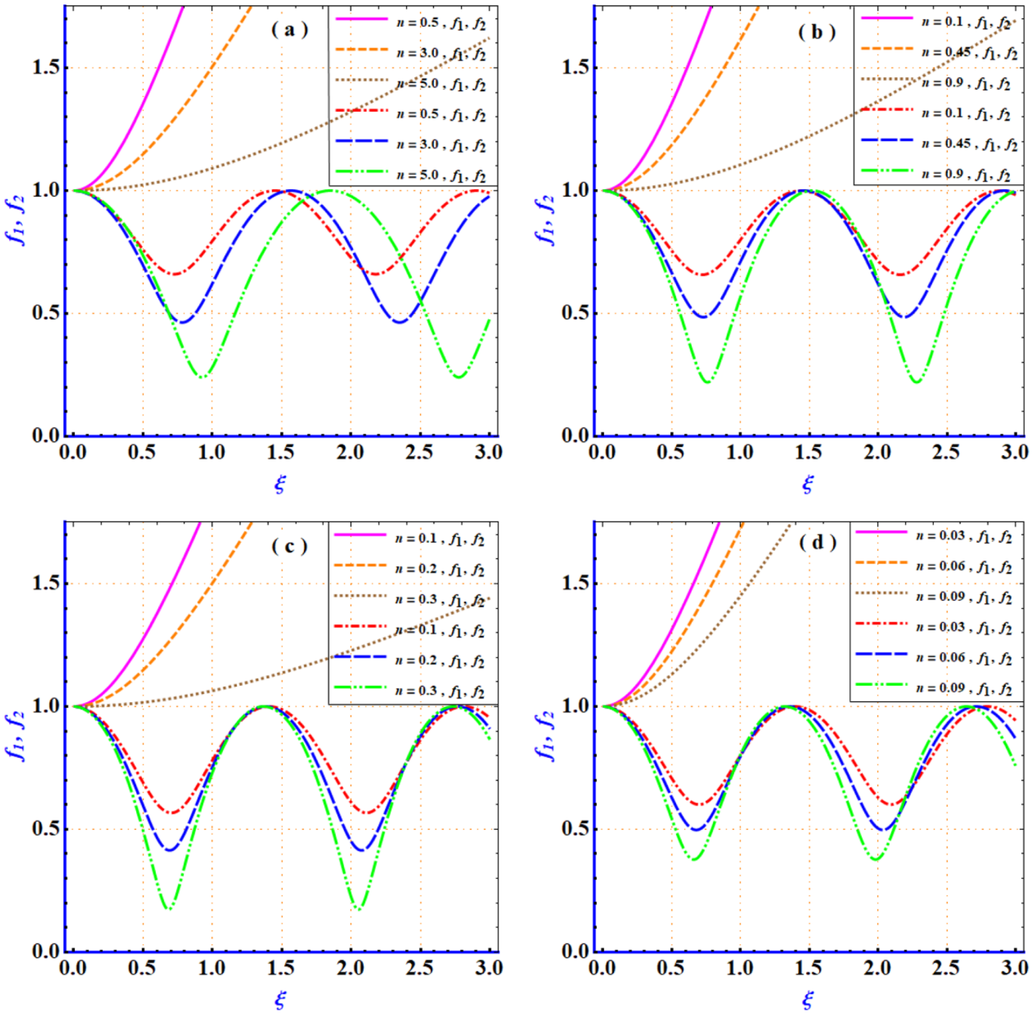

Figure 3. Variation of BWP f_1 and f_2 with ξ with $p_0 = 2.0000$ for distinct skewness parameters s and domains of order n of skew-chG laser beam for $\rho_0 = 3.0000$ (self-focusing), $\rho_0 = 0.5550$ (defocusing), (a) $s = 0.5$ and $0 < n < 5.6277$, (b) $s = 1.0$ and $0 < n < 1.0000$, (c) $s = 1.5$ and $0 < n < 0.3217$ and $s = 2.0$ and $0 < n < 0.1339$.

Table 1. Domains of order n of skew-chG laser beam for a given set of skewness parameter s.

$s = 0.5$	$s = 1.0$	$s = 1.5$	$s = 2.0$
$0 < n < 5.6277$	$0 < n < 1.0000$	$0 < n < 0.3217$	$0 < n < 0.1339$

4. Conclusions

We have explored analytically the domains of the order n of a skew-chG laser beam for a set of skewness parameter s for three delegate characters of beams like self-focusing, defocusing, and self-trapping in homogeneous collisional plasma. Nonlinear and coupled differential equations in transverse dimensions of the beams are established using the parabolic wave equation approach under WKB and paraxial approximations. Due to the symmetry in two transverse directions $n = m$, BWP f_1 and f_2 show perfect synchronization. It is also evident from our study that the critical curves also get affected significantly due to the choice of domain for n. The analytical investigation provides a precise quantitative picture regarding the laser beam propagation. The domain of order n of skew-chG laser beams decreases with an increase in skewness parameter s. In the subcritical region, for a given value of skewness parameter s, the rate of defocusing decreases with increase in the order n of the skew-chG laser beam. In the supercritical region, for a given value of the skewness parameter s, enhanced self-focusing is observed with increase in the order n of the skew-chG laser beam.

Data availability statement

The data that support the findings of the present study are available from the corresponding author upon reasonable request.

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Conflict of interest

The authors have no conflict of interest to disclose.

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