Stronger Self-focusing of Gaussian Laser Beam in Collisionless Plasma Based Exponential Density Profile

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Abstract. The nonlinear features of intense Gaussian laser beams traversing through collisionless plasma along with tangent upward density ramp as well as exponential density profile have been investigated theoretically in the current paper. Naturally, the ponderomotive force is primarily responsible for the collisionless plasma's nonlinear dielectric function. The differential equations for the beam width parameter (BWP) f have been constructed and numerically solved using Akhmanov's parabolic wave equation approach via paraxial and Wentzel-Kramers-Brillouin (WKB) approximations. By utilizing the fourth-order Runge–Kutta method the numerical computation is completed. The noteworthy impact of exponential density profile on propagation dynamics of a Gaussian laser beam is precisely explored and correlated with tangent upward density ramp profile. It is revealed that an exponential density ramp, rather than a tangent upward density ramp, leads the laser beams to become highly focused.

Keywords: Density ramp · Self-focusing · Plasma · Gaussian · Wentzel-Kramers-Brillouin approximation

1 Introduction

Neutral and charged particles combine to form the quasineutral gas known as plasma, which displays collective performance. Self-focusing is a fundamental, 3rd order, and fascinating nonlinear optical phenomenon in which an intense laser beam impacted on a medium modifies the optical characteristics so that the beam comes to focus within the medium. The three key mechanisms that aid to changes in the dielectric function of the plasma in the study of laser-plasma interactions: (i) collisional, (ii) ponderomotive force, and (iii) relativistic. The optical indemnity generated in solids by high-power laser beams is frequently caused by self-focusing [1, 2]. In addition to being of technological interest, the interaction of ultra-high-power laser beam with plasmas is also enriched

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in numerous nonlinear phenomena that are important in several applications, like fast ignition for ICTF (inertial confinement thermonuclear fusion) [3], soft X-ray generation [4], laser driven electron acceleration [5], high harmonic generation [6], ionospheric modification [7], self-phase modulation [8], etc.

In the collisional plasma, local collisional forces, other than collective actions inside it, basically monitor collisional plasma dynamics. Due to the radial distribution of the beams' fields, the electrons in the transverse plane are heated to various temperatures. The power loss from collisions and thermal conduction, along with the Ohmic heating, determine the distribution of electron temperature in the steady state. As a result, the electron density and, consequently, the dielectric constant are redistributed radially [9]. Weak coupling exists in collisionless plasma. Long-range forces rather than collective actions dominate the collisionless plasma dynamics, and coupling between particles because of collisions is often insignificant, with long-range collective interactions binding the particles together. In the plasma the particles interact via Coulomb force, which can traverse a large distance. When the particle interacts with screened Coulomb potential, the long-range component of Coulomb potential is obscured by the combine behavior of the particles. When the laser beam traverses in the collisionless plasma, because of the gradient of heterogeneous irradiance, the ponderomotive force affects the electrons in plasma. Ponderomotive force pushes electrons away from the laser beam's axial portion, where it is more intense. As a consequence of this, the channel experiences an expulsive force that is offset by plasmas pressure from exterior channel, resulting in the density inner side of the channel to reduce. The effective dielectric constant significantly modified in a steady state, and this self-induced inhomogeneity distinguishes the plasma's dielectric function which gives the self-focusing and defocusing of the laser beams [10]. For the purpose of investigating the dynamics of laser beam propagation, the following variety of laser beams profile have been studied in the distinct media: like Gaussian [11–18], Cosh-Gaussian beams [19–22], Hermite–Gaussian beams [23, 24], Hermite-cosh-Gaussian (HChG) beams [25–30], q-Gaussian beams [31–34], Laguerre Gaussian [35], skew-ChG [36–39], Bessel-Gaussian beams [40, 41], and Finite AiG [42] etc., in plasmas has attracted the attention of the many researchers.

It is seems that the density profiles plays very important role in the studies of laser-plasma interactions. Recently, the great interest has been revealed by many researchers towards gradually rising plasma density ramp to prevail over the diffraction of high energy laser beams propagating in the plasma media. A great deal of investigation has been done with various density ramp profiles [43–50], because of its upward growing character; the tangent density ramp profile has been widely explored. But the profile such as $n = n_0 + n_0 tan(\xi/d)$ goes to ∞ at specific value of $\xi = \xi_d$, this means the medium will have an infinite density at ξ_d, as a result, the research of self-focusing via such profiles is restricted upto $\xi < \xi_d$. Such a restriction is effectively solved by introducing exponential density profile i.e. $n = n_0\, exp\, (\xi/d)$, by Sen et al. [44]. Recently, Valkunde et al. [48] has investigated self-focusing of Gaussian laser beams in collisional plasma with exponential density transition as well as they noticed that the self-focusing was enhanced for the exponential density rather than the tangent density ramp profile. However, the current paper explores the comparative study of exponential density and tangent upward density ramp profile in collisionless plasma. The present analysis is studied by using Akhmanov's parabolic wave equation method via paraxial and WKB approximations. Note that for both the density profiles, initially at $\xi = 0$ the density is n_0.

2 Self-focusing

We initiate by taking into account the laser beam propagating through collisionless plasma into the z-axis, the distribution of initial electric field of the laser beams at $z = 0$ is of the form

$$E = A(r, z)exp[i(\omega t - k_0 z)], \tag{1}$$

where $k_0 = \frac{\omega}{c}\sqrt{\varepsilon_0}$, ω is frequency of the laser beam used, c is the speed of light in vacuum and k_0 is wave number in the non-appearance density transition of the plasma.

The following wave equation describes electric field with effective dielectric function ε of plasma medium,

$$\nabla^2 \bar{E} + \frac{\omega^2}{c^2}\varepsilon \ \bar{E} = 0 \tag{2}$$

For cylindrical co-ordinate system, Eq. (2) can also be written as

$$\frac{\partial^2 \bar{E}}{\partial z^2} + \frac{\partial^2 \bar{E}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{E}}{\partial r} + \frac{\omega^2}{c^2}\varepsilon \ \bar{E} = 0 \tag{3}$$

Effective dielectric constant of the plasma [2] is given by

$$\varepsilon = \varepsilon_0 + \phi\left(EE^*\right) \tag{4}$$

with $\varepsilon_0 = 1 - \left(\frac{\omega_p{}^2}{\omega^2}\right)$, where ε_0 and $\phi(EE^*)$ are linear and nonlinear terms of the dielectric function, ω_p is plasma frequency and $\omega_p = \sqrt{4\pi n_e e^2/m}$, where e and m are charge on electron and rest mass of electron. The perturbed density of electron n_e is

$$n_e(\xi) = n_0 \exp\left(\frac{\xi}{d}\right), \tag{5}$$

and

$$n_e(\xi) = n_0 + n_0 \tan\left(\frac{\xi}{d}\right), \tag{6}$$

where $\xi = z/R_d$ dimensionless distance of propagation, $R_d = kr_0^2$, is Rayleigh length with $k = \frac{\omega}{c}\sqrt{\varepsilon}$ wave number in existence of the density transition of plasma, d is the flexible normalizing parameter and n_0 is equilibrium density of electron. Further the plasma frequency becomes

$$\omega_p = \sqrt{\frac{4\pi n_e e^2}{m}} = \omega_{p0}\sqrt{\exp\left(\frac{\xi}{d}\right)} \tag{7}$$

and

$$\omega_p = \sqrt{\frac{4\pi n_e e^2}{m}} = \omega_{p0}\sqrt{\left(1 + \tan\left(\frac{\xi}{d}\right)\right)} \tag{8}$$

The component $\phi(EE^*)$ reveals a saturation behaviour with EE^*. But the saturation analogous to the nonlinearities induced by the nonuniform heating of carriers (collisional plasma) appears at relatively much lower wave intensities (*M/m* times lower) than the case of collisionless plasma. Intensity dependent dielectric function for the collisionless plasma [2, 23, 25] is,

$$\phi(EE^*) = \frac{{\omega_p}^2}{\omega^2}\left[1 - exp\left(-\frac{3m\alpha EE^*}{4M}\right)\right] \tag{9}$$

with $\alpha = \left(\frac{e^2 M}{6k_B T_0 \omega^2 m^2}\right)$,

where, M, T_0 and k_B, are mass of ion, critical temperature of plasma and Boltzmann constant respectively. Substituting for $\overline{\mathrm{E}}$ and ε from Eqs. (1) and (4) in Eq. (3), one can get

$$\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r} + \frac{\omega^2}{c^2}\phi(EE^*)A = 2ik_0\frac{\partial A}{\partial z} \tag{10}$$

The Eq. (10) is well known parabolic wave equation which represents evolution of the beam envelope in the plasma. Now we can express A as follows

$$A(r,z) = A_0(r,z)\exp[-ikS(r,z)] \tag{11}$$

Where, $A_0(r,z)$and$S(r,z)$ are real functions of r and z and S is the eikonal of the wave. Putting Eq. (11) for A in Eq. (10) and isolating the imaginary and real components, one can get

$$2\left(\frac{\partial S}{\partial z}\right) + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{1}{k^2 A_0}\left(\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r}\right) + \frac{\phi}{\varepsilon_0}\left(A_0 {A_0}^*\right) \tag{12}$$

and

$$\frac{\partial A_0^2}{\partial z} + \left(\frac{\partial S}{\partial r}\right)\left(\frac{\partial A_0^2}{\partial r}\right) + \left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right)A_0^2 = 0 \tag{13}$$

By following paraxial method specified by Akhmanov et al. [1] and subsequently extended by Sodha et al. [2], the solutions of Eqs. (12) and (13) for Gaussian laser beams is given by

$$S = \frac{r^2}{2}\beta(z) + \phi(z), \tag{14}$$

And

$$A_0^2 = \frac{{E_0}^2}{f^2}\exp\left(-\frac{r^2}{f^2 r_0^2}\right), \tag{15}$$

where f is BWP and r_0 is the initial beam radius of the laser beams. By following the paraxial method specified by Akhmanov et al. [1] and pedagogical simple extensions by Sodha et al. [2], we obtain in the existence of tangent density ramp profile the nonlinear, ordinary, second-order differential equation for BWP as follows [1, 2],

$$\left\{\left(1-\frac{\xi\sec\left(\frac{\xi}{d}\right)^{2}\omega_{p0}^{2}}{2d\omega^{2}\left[1-\frac{\omega_{p0}^{2}}{\omega^{2}}\left(1+\tan\left(\frac{\xi}{d}\right)\right)\right]}\right)\frac{d^{2}f}{d\xi^{2}}\right\}= \left\{\begin{array}{c}\left[\left(\frac{1}{f^{3}}\right)-\left(\frac{3m\alpha E_{0}{}^{2}\omega_{p0}^{2}r_{0}^{2}\left(1+\tan\left(\frac{\xi}{d}\right)\right)\exp\left(-\frac{3m\alpha E_{0}{}^{2}}{4Mf^{2}}\right)}{4Mf^{3}c^{2}}\right)\right]+\\ \left[\frac{\omega_{p0}^{2}\sec\left(\frac{\xi}{d}\right)^{2}}{2d\omega^{2}\left[1-\frac{\omega_{p0}^{2}}{\omega^{2}}\left(1+\tan\left(\frac{\xi}{d}\right)\right)\right]}\left(1-\frac{\xi\sec\left(\frac{\xi}{d}\right)^{2}\omega_{p0}^{2}}{2d\omega^{2}\left[1-\frac{\omega_{p0}^{2}\left(1+\tan\left(\frac{\xi}{d}\right)\right)}{\omega^{2}}\right]}\right)\left(\frac{df}{d\xi}\right)\right]-\\ \left[\left(\frac{\xi\sec\left(\frac{\xi}{d}\right)^{2}\omega_{p0}^{2}}{2d\omega^{2}f\left[1-\frac{\omega_{p0}^{2}}{\omega^{2}}\left(1+\tan\left(\frac{\xi}{d}\right)\right)\right]}\right)\left(\frac{df}{d\xi}\right)^{2}\right]\end{array}\right\} \tag{16}$$

Similarly, in the existence of exponential density ramp profile, we obtain the nonlinear, ordinary, second differential equation for BWP as follows [1, 2],

$$\left\{\left(1-\frac{\xi\omega_{p0}^{2}\exp\left(\frac{\xi}{d}\right)}{2d\omega^{2}\left[1-\frac{\omega_{p0}^{2}}{\omega^{2}}\exp\left(\frac{\xi}{d}\right)\right]}\right)\frac{d^{2}f}{d\xi^{2}}\right\}= \left\{\begin{array}{c}\left[\left(\frac{1}{f^{3}}\right)-\left(\frac{3m\alpha E_{0}{}^{2}\omega_{p0}^{2}r_{0}^{2}\exp\left(-\frac{3m\alpha E_{0}^{2}}{4Mf^{2}}+\frac{\xi}{d}\right)}{4Mf^{3}c^{2}}\right)\right]+\\ \left[\frac{\omega_{p0}^{2}\exp\left(\frac{\xi}{d}\right)}{2d\omega^{2}\left[1-\frac{\omega_{p0}^{2}}{\omega^{2}}\exp\left(\frac{\xi}{d}\right)\right]}\left(1-\frac{\exp\left(\frac{\xi}{d}\right)\xi\omega_{p0}^{2}}{2d\omega^{2}\left[1-\frac{\exp\left(\frac{\xi}{d}\right)\omega_{p0}^{2}}{\omega^{2}}\right]}\right)\left(\frac{df}{d\xi}\right)\right]-\\ \left[\left(\frac{\xi\omega_{p0}^{2}\exp\left(\frac{\xi}{d}\right)}{2d\omega^{2}f\left[1-\frac{\omega_{p0}^{2}}{\omega^{2}}\exp\left(\frac{\xi}{d}\right)\right]}\right)\left(\frac{df}{d\xi}\right)^{2}\right]\end{array}\right\} \tag{17}$$

3 Results and Discussion

The second-order, nonlinear, ordinary differential Eqs. (16) and (17) indicate that the evolution of the BWP f with respect to dimensionless propagation distance ξ for the tangent as well as exponential density ramp profile, respectively. The 1st term on the R.H.S of Eqs. (16) and (17) gives diffraction divergence, and is accountable for defocusing, while the 2nd term gives nonlinear convergence, which is accountable for self-focusing and left over terms arises because of the plasma density ramp. The dominant factor affecting the laser beam is diffraction, which is more significant than self-focusing due to medium absorption and scattering losses. When a laser beam achieves perfect balance between self-focusing and diffraction, it propagates in the self-trapped mode. The preceding Eqs. (16) and (17) are numerically solved by selecting the appropriate laser-plasma parameters: $r_0 = 20 \times 10^{-4}$ cm, $\lambda = 1.06$ μm, $n_0 = 3.9645 \times 10^{17}$ cm^{-3}, $\omega = 1.7760 \times 10^{15}$ rad/s, $\alpha E_0{}^2 = 600$, $d = 10$ and the intensity of the laser beam being used is $I \sim 5.7 \times 10^{17}$ W/cm^2. It is crucial to mention here that in the non-appearance of both the density transitions ($\xi = 0$ means $n = n_0$ i.e. uniform density) the Eqs. (16) as well as (17) becomes

$$\frac{d^2 f}{d\xi^2} = \frac{1}{f^3} - \frac{3m\alpha E_0{}^2 \omega_{p0}^2 r_0^2}{4Mc^2 f^3} \tag{18}$$

The Eq. (18) is analogous to an identical equation achieved initially by Sodha et al. [2] for the Gaussian beams propagating in collisionless plasma along with uniform density.

To exhibit explicitly the variation of exponential $n_e = n_0 \exp(\xi/d)$ as well as tangent profile $n_e = n_0 + n_0 \tan(\xi/d)$, we have plotted the comparative density of electron n/n_0 as a function of dimensionless propagation distance ξ. Figure 1 illustrates both exponential as well as tangent density ramp profiles for $n_0 = 3.9645 \times 10^{17}$ cm^{-3} and $d = 10$. Since Fig. 1 it is evident that the tangent function rises promptly with rising ξ and shoot to infinity for specific value of i.e. $\xi = \xi_d$. For current problem $\xi_d = 15.8$. Conversely, the exponential profile depicts smooth change in density with dimensionless propagation distance ξ.

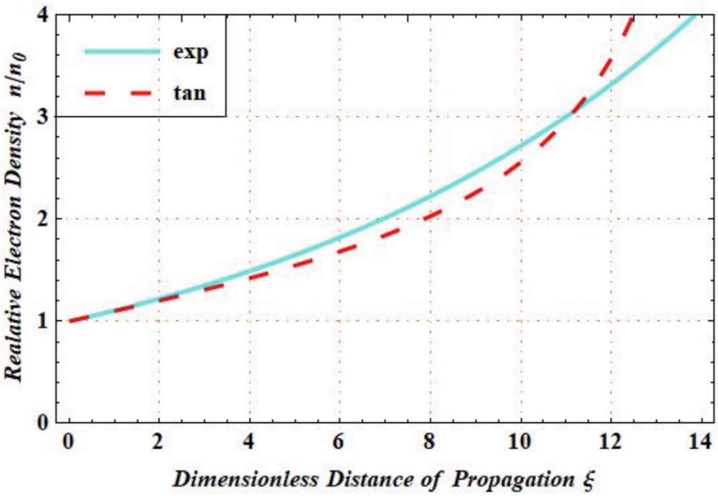

Fig. 1. Variation of n/n_0 versus ξ for two distinct profiles $n = n_0 + n_0 tan(\xi/d)$, $n = n_0 exp(\xi/d)$.

To represent explicitly the variation of BWP f as well as intensity dependent dielectric function ϕ with respect to the normalized distance of propagation ξ, we have plotted f as well as ϕ as a function of ξ. Figure 2, describes the correlation between BWP f versus ξ for tangent density ramp, exponential density ramp and plasma with no ramp. When the density of the plasma is homogenous, periodic self-focusing is noticed and the oscillatory self-focusing by shrinking in spot size along with ξ is noticed in the existence of density ramp. It is recognized that for exponential ramp enhanced self-focusing is seen rather than tangent ramp profiles. The cause of such occurrence is revealed in Fig. 3. Also the intensity dependent nonlinear dielectric function ϕ with respect to dimensionless propagation distance ξ is illustrated Fig, 3. The nonlinear dielectric constant ϕ rises quickly in the existence of ramp profiles in comparison with the ϕ without the ramp profile. As the laser beam enters deeper into plasma, the power of self-focusing improves, which causes the laser spot size shrinks gradually and become more focused. From Figs. 2 and 3, it is seen that, the lowest f value corresponds highest ϕ value and vice versa.

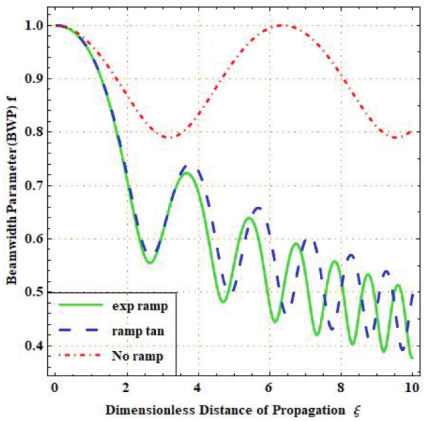

Fig. 2. Variation of BWP versus ξ.

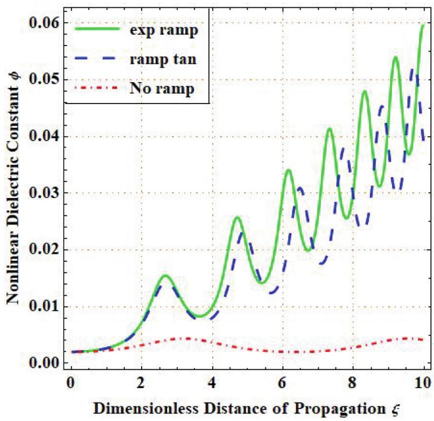

Fig. 3. Variation ϕ versus ξ.

The investigation of self-focusing was restricted by the divergence of the tangent profile for specific value of propagation distance ξ. Such a restriction is effectively resolved by introducing a new kind of density profile namely exponential density profile i.e. $n = n_0 \exp(\xi/d)$. Exponential density is further better desirable than that of density profile of tangent ramp, for enormous value of ξ. Thus, by gradually raising the density, the density profile of exponential ramp gradually enhances the self-focusing. As the laser travels over the exponential density ramp zone, the channel gradually shrinks. The laser spot size oscillation amplitude normally decreases in plasma environment, though its frequency increases. The laser beam becomes further focused across many Rayleigh lengths as a result of the exponential plasma density ramp.

Figure 4, represents the normalized intensity I/I_0 versus normalized propagation distance ξ and normalized beam radius r/r_0 in collisionless plasma. It depicts how the intensity of the laser beams improves as it passes into the collisionless plasma, illustrating self-focusing. For the spot size profile the density plot is presented in Fig. 5 to support this conclusion. Also from Fig. 5, one can notice that with increasing ξ the spot size

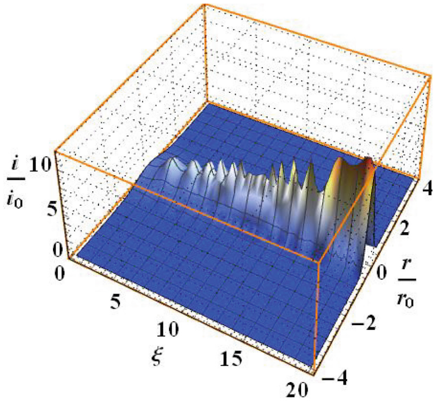

Fig. 4. 3D intensity profile of normalized intensity I/I_0 Vs normalized beam radius r/r_0 and dimensionless distance of propagation ξ.

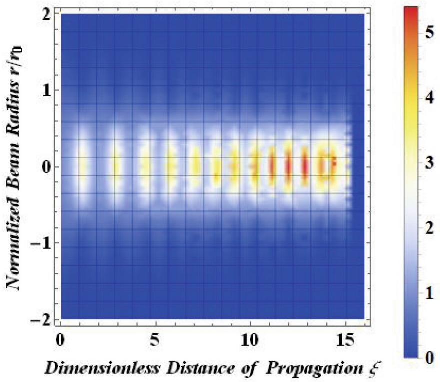

Fig. 5. Density profile of normalized intensity I/I_0 versus r/r_0 and ξ.

reduces and that is responsible to increasing the intensity of laser beam. The laser beams intensity peaks correlate to valleys of BWP f.

4 Conclusion

Through the use of the parabolic wave equation method and paraxial and WKB approximations, we studied how Gaussian laser beams are affected by exponential and tangent density ramp profiles when travelling through collisionless plasma. We found that applying both types of ramp profiles resulted in a stronger self-focusing of the laser beam in the collisionless plasma. However, the self-focusing was even stronger with the exponential density ramp profile as compared to the tangent density ramp profile. Our investigation indicates that the exponential density ramp profile is more desirable for propagating a laser beam over many Rayleigh lengths. These findings have significant implications for various laser-plasma applications such as laser particle accelerator, laser-driven fusion, inertial confinement fusion etc.

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