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M.S.C.

M.Sc. (Part - I) (Semester - II) Examination, 2013

STATISTICS (Paper - VI) (Credit System) (A.F.)

Probability Theory

Sub. Code : 42327

Day and Date : Monday, 15 - 04 - 2013

Total Marks : 80

Time : 11.00 a.m. to 2.00 p.m.

- Instructions : 1) Question No. 1 is compulsory.
2) Attempt any 4 questions from No.2 to 7.
3) Figures to the right indicate marks to the sub-question.

Q1) Answer any eight of the following :

[8 × 2 = 16]

a) Define :

i) σ -field

ii) σ -field

Give an example of σ -field.

b) Define :

i) $\limsup A_n$

ii) $\liminf A_n$

iii) $\lim A_n$

where A_n is a sequence of sets.

c) Define :

i) probability measure

ii) generalized probability measure

Give an example of each.

d) Show that an indicator function is measurable.

e) Define :

i) simple function

ii) random variable

f) State monotone convergence theorem.

g) Define :

i) Convergence in probability

ii) Almost sure convergence

h) Define a characteristic function and state any two properties.

i) State Lindeberg-Feller theorem on CLT.

j) Give an application of CLT.

Q2) a) Let $A_n = A$ if $n = 1, 3, 5, \dots$ and $A_n = B$ if $n = 2, 4, 6, \dots$

Obtain $\overline{\lim} A_n, \underline{\lim} A_n$. Does $\lim A_n$ exist? Justify your answer.

[6]

b) Show that $\underline{\lim} A_n \subseteq \overline{\lim} A_n$

[10]

- Q3) a) Define minimal σ -field and give an example of the same. [4]
 b) Show that the intersection of arbitrary number of fields is also field. [6]
 c) Show that every field contains the empty set ϕ and the whole space Ω . Is a set containing ϕ & Ω always field? Justify your answer. [6]

- Q4) a) Define inverse mapping and show that inverse mapping preserves all set relations. [10]
 b) If A_1, A_2 are measurable sets and a function X is defined by, [6]

$$X(W) = \begin{cases} -1 & W \in A_1 \\ +1 & W \in A_1^c A_2 \\ 0 & W \in A_1^c A_2^c \end{cases}$$

Examine whether X is measurable.

- Q5) a) Show that a random variable X is a finite limit of a sequence of simple random variables. [6]
 b) If $A_n \rightarrow A$ then show that $P(A_n) \rightarrow P(A)$. [6]
 c) Define expectation of a random variable. Give an example of a random variable for which expectation does not exist. [4]

- Q6) a) Prove: $X_n \xrightarrow{P} 0$ iff $E \frac{|X_n|}{1+|X_n|} \rightarrow 0$ as $n \rightarrow \infty$. [8]

- b) Prove: $X_n \xrightarrow{\text{a.s.}} X$ iff as $n \rightarrow \infty$

$$P\left(\bigcup_{k=n}^{\infty} \left[W : |X_k - X| \geq \frac{1}{r}\right]\right) \rightarrow 0 \quad \forall r, \text{ an integer} \quad [8]$$

- Q7) Write short notes on any four of the following: [4 × 4]

- a) Weak and strong law of large numbers.
 b) Borel-Cantelli Lemma.
 c) Yule Slutsky results.
 d) Inversion theorem.
 e) Kolmogorov three series criterion.
 f) Dominated convergence theorem.



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M.Sc. (Part - I) (Semester - II) Examination, April - 2014
STATISTICS (Paper - VI) (Revised) (CBCS)

Probability Theory
Sub. Code : 61313

Day and Date : Wednesday, 09-04-2014

Total Marks : 80

Time : 11.00 a.m. to 2.00 p.m.

- Instructions :**
- 1) Question number 1 is compulsory.
 - 2) Attempt any four questions from question No. 2 to 7.
 - 3) Figures to right indicate marks to the questions.

Q1) Answer the following sub questions.

[16]

- a) Define a field and give an example of a field.
- b) Comment on a statement - Every field must contain null set.
- c) Define a σ -field.
- d) If $A_n = A \forall n \geq 1$ then what is $\lim A_n$?
- e) Define $\liminf A_n$ and $\limsup A_n$, when A_n is a sequence of sets.
- f) Define inverse mapping?
- g) What is simple function?
- h) What is indicator function?
- i) What is an additive property of probability measure?
- j) Give an example of a simple random variable.
- k) State weak law of large number.
- l) Define characteristic function and state a property of it.
- m) Define an expectation of a non-negative random variable.

n) State inversion theorem of characteristic function.

o) Define independence of two random variables.

p) State Lindeberg Feller CLT.

Q2) a) Let a sequence of sets $\{A_n\}$ be defined as $A_n = \left[0, 1 + \frac{3}{n}\right], n \geq 1$

Obtain $\lim A_n$, if exists.

[8]

b) State and prove relationship between $\liminf A_n$ and $\limsup A_n$.

[8]

Q3) a) Define (i) Probability measure (ii) Conditional probability measure.

Let $A = \{A, A^c, \Omega, \phi\}$ construct a suitable probability measure on A. [8]

b) If $A_n \uparrow A$ then prove that $P(A_n) \rightarrow P(A)$. Comment on similar result for sequence A_n^c .

[8]

Q4) a) In usual notations, prove that $\sigma\{x^{-1}(E)\} = x^{-1}\{\sigma(E)\}$

[8]

b) Let $\Omega = \{-2, -1, 0, 1, 2\}$. If $x(i) = i, i = 0, \pm 1, \pm 2$, obtain σ -fields induced by x and x^2 .

[8]

Q5) a) Define almost sure convergence and convergence in probability.

Let $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$ then prove that $X_n + Y_n \xrightarrow{P} X + Y$.

[8]

b) State and prove monotone convergence theorem.

[8]

- Q6) a) Obtain characteristic function of standard normal variate. [6]
b) State Kolmogorov's three series theorem. [4]
c) Give an application of dominated convergence theorem. [6]

Q7) Write short notes on the following :

[4 × 4 = 16]

- a) Borel - Cantelli Lemma.
b) Convergence in r^{th} mean.
c) Convergence in distribution.
d) Lebesgue - Stieltjes measure.



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M.Sc. (Part - I) (Semester - II) Examination, April - 2015

STATISTICS (Paper - VI)

Probability Theory (CBCS)

Sub. Code : 61313

Day and Date : Monday, 06-04-2015

Total Marks : 80

Time : 10.30 a.m. to 01.30 p.m.

- Instructions :
- 1) Question No. 1 is compulsory.
 - 2) Attempt any four questions from Question No. 2 to 7.
 - 3) Figures to right indicate marks to the questions.

Q1) Answer the following :

[16]

- a) Define an indicator function.
- b) Define characteristic function.
- c) State any two properties of characteristic function.
- d) State Kolmogorov's three series theorem.
- e) Define measurable space.
- f) Give an example of a measurable space.
- g) Give an example of a simple random variable.
- h) Define conditional expectation.
- i) Define distribution function of a random variable.
- j) State Yule-Slutsky result.
- k) State Lindberg-Feller Theorem on CLT.
- l) Comment on the statement: Every field must contain ϕ and Ω .
- m) Comment on the statement: Indicator function is a random variable.

- n) State weak law of large numbers.
- o) Define Borel field.
- p) Define counting measure.

Q2) a) Define a Monotone field. Show that σ -field is a monotone field. Is converse true? Justify. [8]

b) Define inverse mapping and show that inverse mapping preserves all set relations. [8]

Q3) a) Define [8]

i) Lebesgue-Stieltje's Measure

ii) Probability measure

iii) Field

iv) σ -field

b) Define random variable. Is Borel function of a random variable also a random variable? Justify. [8]

Q4) a) Find $\lim A_n$ if exists for the following sequence of sets: [8]

i)
$$A_n = \begin{cases} A & \text{if } n \text{ is even} \\ B & \text{if } n \text{ is odd} \end{cases}$$

ii)
$$A_n = \left(1 + \frac{1}{n}, 2 + \frac{1}{n}\right)$$

b) Discuss [8]

i) Bernoulli random variable

ii) Poisson random variable

- Q5) a) State and prove dominated convergence theorem. [8]
b) Define convergence in distribution and convergence in probability. Show that convergence in probability implies convergence in distribution. Is converse true? Justify. [8]
- Q6) a) State and prove Borel-Cantelli lemma. [6]
b) Explain the terms with illustrations. [6]
i) Mutual Independence.
ii) Pairwise Independence.
c) Give an example of a mapping which is not a random variable. Justify. [4]
- Q7) Write short notes on the following : [4 × 4 = 16]
a) Fatou's lemma.
b) Almost sure convergence.
c) Continuity property of probability measure.
d) Probability measure induced by random variable X.

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No.

M.Sc. (Part - I) (Semester - II) (C.B.C.S.)

Examination, April - 2016

STATISTICS

Probability Theory (Paper - VI)

Sub. Code : 61313

Day and Date : Friday, 01 - 04 - 2016

Total Marks : 80

Time : 11.00 a.m. to 2.00 p.m.

- Instructions :
- 1) Question No. 1 is compulsory.
 - 2) Attempt any four questions from Question No. 2 to 7.
 - 3) Figures to right indicate marks to the questions.

Q1) Attempt the following:

[16 × 1 = 16]

- a) Define Borel σ -field.
- b) Define limit of monotone increasing sequence of sets.
- c) Examine a class of finite and cofinite sets to be a field.
- d) Give an example of a field which is not monotone.
- e) Let $\Omega = \mathbb{R}$ for $X(\omega) = |\omega|$, write down $X^{-1}(B)$ for $B \in \mathcal{R}$.
- f) Prove or disprove : $I(A^c) = 1 - I(A)$.
- g) Give an example of a mapping which is not a random variable.
- h) In usual notations, show that $P(\emptyset) = 0$.
- i) Write down the Lebesgue measure of set A , where A is set of all rationals in $(0, 1)$.
- j) Define Lebesgue - steiltje's measure.

P.T.O.

- k) State Liapouov's form of CLT.
- l) State Borel-Cantelli Lemma.
- m) Prove or disprove: If $X \geq 0$ a.s. then $E X \geq 0$.
- n) Define pairwise independence.
- o) State Kolmogorov's three series theorem.
- p) In usual notations, show that $|\phi(t)| \leq 1$.

$b(t) \leq 1$
 $b(0) = 1$

- Q2) a) Explain limit of arbitrary sequence of sets. Obtain limit of the sequence of sets defined by

$$\left. \begin{aligned} A_{2n} &= \left(0, \frac{1}{2n}\right) \\ A_{2n+1} &= \left[-1, \frac{1}{2n+1}\right] \end{aligned} \right\} n=1, 2, \dots$$

if exists.

- b) Define σ -field with example.

Prove or disprove : An arbitrary intersection of σ -fields is also a σ -field.

[8 + 8]

- Q3) a) Show that inverse mapping preserves set relations.

- b) Explain probability measure induced by a random variable X . Obtain Binomial random variable through usual probability measure P and corresponding induced probability measure.

[8 + 8]

- () a) Explain expectation of an arbitrary random variable. Show that X is integrable iff $|X|$ is integrable.
- b) State and prove monotone convergence theorem.

[8 + 8]

- Q a) Define Borel function. Show that Borel function of a random variable is also a random variable.
- b) Explain Lindberg-Feller theorems on CLT. Give one application of the same.

[8 + 8]

- Q a) Prove or disprove : $X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{D} X$.
- b) Prove or disprove : $X_n \xrightarrow{a.s.} X \Rightarrow X_n \xrightarrow{r} X$.
- c) Obtain characteristics function of $N(\mu, \sigma^2)$.
- d) Let B_n be a non-decreasing sequence of sets belonging to \mathcal{F} and (Ω, \mathcal{F}, p) be the probability space. Then show that

$$P(B_n) \uparrow P(B), \text{ where, } B = \bigcup_{n=1}^{\infty} B_n.$$

[4 × 4]

- Q Write short notes on the following:

[4 × 4]

- a) Dominated convergence theorem.
- b) Characteristic function and its properties.
- c) Indicator function and its properties.
- d) Economical definition of a random variable.

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M.Sc. (Part - I) (Semester - II) (Credit System) Examination, November - 2014

STATISTICS (Paper - VI)

Probability Theory (A.F)

Sub. Code : 42327

Day and Date : Monday, 10 - 11 - 2014

Total Marks : 80

Time : 02.30 p.m. to 5.30 p.m.

- Instructions :
- 1) Question No. 1 is compulsory.
 - 2) Attempt any 4 questions from No. 2 to 7.
 - 3) Figures to right indicate marks.

Q1) Answer any eight of the following : [8 × 2 = 16]

- a) Define field and σ - field. Give an example of a field which is not a σ - field.
- b) Show that the limit of a monotone sequence of sets exists.
- c) If $\{A_n\}$ is a sequence of events on probability space (Ω, A, P) , Show that $P\left(\bigcap_{n=1}^{\infty} A_n\right) = 1$, when $P(A_n) = 1, \forall n$.
- d) Define Lebesgue - steiltje (LS) measure and state relationship between LS measure and probability measure.
- e) Define Lim inf and lim sup of a sequence of sets $\{A_n\}$. Find $\lim A_n$ if $A_n = \left(2 - \frac{1}{n}, 3 + \frac{1}{n}\right)$.
- f) Define convergence in distribution of a sequence of random variables.
- g) Define a monotone field.
- h) State Kolmogorov zero - one law. Give its application.
- i) Let A, B, C be three events. Define pairwise independence and mutual independence of these events.
- j) State Khintchine's WLLN.

Q2) a) Prove that the intersection of arbitrary number of fields is a field. [8]

b) Examine whether the sequence. $A_n = \left\{ w : 0 < w < b + \frac{(-1)^n}{n} \right\}$ is convergent or not, given $b > 1$. [8]

Q3) a) Define a probability space (Ω, A, P) , if $\{A_n\}$ is sequence of events such that $\lim A_n$ exists, prove that $P\left(\lim_{n \rightarrow \infty} A_n\right) = \lim_{n \rightarrow \infty} P(A_n)$. [8]

b) Define a random variable. If x_1, \dots, x_n are random variables, show that $x_1 + x_2, \max x_j, 1 \leq j \leq n$ are also random variables. [8]

Q4) a) Define Characteristic function of a random variable obtain the same for normal distribution. [8]

b) Using inversion formula, obtain the distribution whose characteristic function is $Q(t) = e^{-|t|}$. [8]

Q5) a) State and prove Borell - cantelli lemma. [12]

b) Show that if $|X_n| \leq y$, y integrable, then

$$X_n \xrightarrow{a.s.} X \Rightarrow E(X_n) \rightarrow E(X) \quad [4]$$

Q6) a) State Lindberg - feller and Liapunov form of central limit theorem. (CLT) Give an application of CLT. [8]

b) Establish relationship between convergence in probability and almost sure convergence. [8]

Q7) Write short notes on any four [4 × 4]

- Borel field
- Probability measure and countable measure.
- Limits of sequence of random variables
- Dominated convergence theorem.
- Yule - Shitsky results.
- Convergence of sequence in distribution.

