Shivaji University, Kolhapur

M.Sc (Statistics/Applied Statistics and Informatics) Exam

Question Bank

Course: Distribution Theory (74909/83440/74976)

Short answer questions (2 marks)	
1	Define: cumulative distribution function
2	State the properties of cumulative distribution function.
3	Define quantiles.
4	Define mixture of distributions.
5	Let F(x) be a cumulative distribution function (cdf). Verify whether $[F(x)]^{\alpha}$, $\alpha > 0$ is a cdf or not?
6	Show that the exponential distribution has memoryless property.
7	Give the real life application of normal distribution.
8	State the forget fullness property of exponential distribution.
9	Define location family. Give one example of the same.
10	Define scale family. Give an example.
11	Define shape family. Give an example.
12	Suppose X has $U(0, 1)$ distribution, find the distribution of $Y = -\log X$
13	Let X be distributed as U (0, θ). Then find distribution of $Y = X/\theta$
14	Let X be distributed as (n, p). find distribution of $Y = n - X$
15	State Holder's inequality.
16	State Markov inequality.
17	State Jensen's inequality.
18	State Minkowski inequality.
19	Illustrate Tchebysheff inequality
20	State the application of the Tchebysheff inequality
21	Illustrate probability integral transformation.
22	Suppose $U \sim x^2{}_n$ and V~ N (0,1) then find E (U+V) and Var(U+V)
23	If X is standard normal variate then find mean and variance of X^2
24	If 0 < a < 1 and p.m.f. of random variable X is P (x) = k a^x, x = 0, 1, 2,... then find P(X = 0).
25	If X is degenerate random variable at β then find E (X) and V(X).
26	If X is symmetric about α then find the symmetric point of $(X-\alpha)$.
27	Let X be a $N(\mu, \sigma^2)$ variable. Then state the mean of $Y=e^x$.
28	Let X be a Weibull (a, b) distributed random variable obtain E (X).
29	If X is Cauchy (0, 1) then state mean of Y=1/X ?
30	If X is standard exponential random variable then find the distribution of e^{-X}
31	Define Bivariate Normal distribution.
32	Define Marshall-Olkin bivariate exponential distribution.
33	State the forget fullness property of exponential distribution.
34	Let (X,Y) follows bivariate normal distribution then show that X and Y are independently distributed if Corr(X,Y)=0.
35	Illustrate Independence of random variables with suitable example.
36	Define a compound distribution. Obtain its expected value.
37	Obtain the pdf of convolution of two independent exponential random variables.

38	State Fisher Cochran theorem.
39	Define non-central t distribution.
40	Define non-central F distribution.
41	Define non-central Chi-square.
Short notes (4 marks)	
1	Properties of a Distribution Function.
2	Decomposition of mixed type distribution functions.
3	Mixture of probability distributions.
4	Probability Integral transform and its applications.
5	Symmetric distribution and their properties.
6	Applications of moment inequalities.
7	Jensen inequality and its applications.
8	Applications of moment inequalities
9	Location and scale family.
10	Transformations of random variables.
11	Random vector and its joint distribution.
12	Independence
13	Joint MGF of random vector
14	Transformation of bivariate random variables.
15	Bivariate normal distribution.
16	Marshall-Olkin bivariate exponential distribution.
17	Convolution of two random variables.
18	Compound distributions.
19	Fisher-Cochran theorem and its applications.
20	Non-central F distribution.
21	Non-central t distribution.
22	Non-central chi-square distribution.
Long answer questions (8 Marks each)	
1	Describe the algorithm of decomposition of a mixed type cumulative distribution function F(.) in to discrete and continuous components. Also explain how you will obtain it's the raw moments.
2	Define cumulative distribution function (CDF). State its properties. Check whether following function is a CDF or not? $$F(x) = \begin{cases} 0 & if\ x \leq 1, \\ 1 - \frac{1}{x} & if\ x > 1. \end{cases}$$
3	Decompose the following distribution function into discrete and continuous components. $$F(x) = \begin{cases} x & if\ x < 0, \\ x + \frac{1}{2} & if\ 0 \leq x < 1, \\ 1 & if\ x \geq 1. \end{cases}$$

4	Decompose the following cdf $F_X(x)$ into discrete and continuous components. $$F_X(x)=\begin{cases} 0 & ;x<0 \\ \frac{1}{4}+\frac{x}{4} & ;0\le x<1 \\ \frac{1}{2}+\frac{x}{4} & ;1\le x<2 \\ 1 & ;x\ge 2 \end{cases}$$ Hence, compute E(X) and V(X) of X.
5	Consider $$F(x)=\begin{cases} 0 & \text{if } x<-2 \\ 1/3 & \text{if } -2\le x<0 \\ 1/2 & \text{if } 0\le x<5 \\ 1/2+(x-5)^2/2 & \text{if } 5\le x<6 \\ 1 & \text{if } x\ge 6 \end{cases}$$ Decompose F(x) as a mixture of discrete and continuous distribution functions. Find mean and variance of X.
6	Verify that, following function $$F(x)=\begin{cases} 0 & \text{if } x<-2 \\ 0.25 & \text{if } -2\le x<2.6 \\ 0.36 & \text{if } 2.6\le x<3.8 \\ 0.48 & \text{if } 3.8\le x<8 \\ 0.7 & \text{if } 8\le x<11 \\ 1 & \text{if } x\ge 11 \end{cases}$$ is a distribution function of some r.v. X and find the associated probability distribution of X.
7	The random variable X has following pdf $$f(x)=\begin{cases} \frac{3x^2}{2} & if\ \lvert x\rvert<1, \\ 0 & otherwise. \end{cases}$$ Then, find (i) $E(\lvert X\rvert)$ (ii) $V(\lvert X\rvert)$.
8	State and prove the relation between distribution function of a continuous random variable and uniform random variate.
9	State and prove: moment inequality. Give its application.
10	State and prove Holder's inequality. Give its application.
11	State and prove Minkowski inequality. Give its application.
12	State and prove Markov's inequality. Give its application.
13	State and prove Jensen's. Give its application.

14	State and prove Markov inequalities. Give its application.
15	Using Jensen's inequality derive the relation between AM, GM and HM.
16	Let X be a random variable having symmetric distribution, symmetric about θ. Show that E(X) = θ & Median (X) = θ.
17	Let F(x) be a distribution function of a random variable X. Examine whether $[F(x)]^2$ and 1 – F(x) are distribution functions.
18	Let X ~U (-2,2). Obtain the distribution of X^n (n is positive integer) and the positive part of X.
19	Suppose X has Gamma (a, b), obtain the distribution of Y=2X.
20	Let X be a non-negative continuous random variable with distribution function F (X). Show that $E(X) = \int_0^{\infty}[1 - F(x)]dx$
21	Let X is Poisson (λ) and Y is Gamma (μ+1), Then show that P(X<= μ)=P(Y>=λ).
22	Let X has Exponential(mean=θ). Find the distribution of (i) $y = \frac{x}{\theta}$ (ii) $z = \left(1 - e^{-\left\{\frac{x}{\theta}\right\}}\right)$
23	Define scale family of distribution. Also examine whether U (0, θ) and N(0, σ^2) are members of scale family or not?
24	Define location-scale (L-S) family of distributions. Express the quantiles of a L-S family member in terms of those of the standard distribution. Give an example of it. Give an example of a distribution which is not a member of this family.
25	Assume (X, Y) has trinomial distribution, find the conditional expectation of Y given X = x.
26	Define convolution of two random variables. Let X and Y be independent standard exponential random variables. Find the p.d.f. of X-Y using convolution.
27	Define a symmetric distribution. Examine whether the t distribution with n degrees of freedom is symmetric. Give an example (with justification) of a distribution that is not symmetric.
28	Assume (X, Y) has trinomial distribution, find the conditional expectation of Y given X = x.
29	Let (X, Y) ~ N $(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Obtain the conditional distribution of Y given X.
30	A fair coin is tossed 3 times. Let X = number of heads in 3 tosses and Y = absolute difference between number of heads & tails. Find joint and marginal distribution of X and Y.
31	Let random variable X has exponential distribution with mean θ. Find the distribution of Y = X / θ.
32	Let random variable X has exponential distribution with mean β. Find the distribution of $Y = X^{1/\beta}$, β> 0.
33	Let X is non-negative random variable with P(X = x)=P_x , x = 0, 1, 2,…. Show that E(X) = $\sum_0^{\infty} P(X > x)$.
34	If X has U(0,1) then derive the distribution of Y=a +(b-a)X, Z= –θ log(1-X) and W=1-X.
35	If X has exponential with mean θ. Derive the distribution of random variables $Y = 1 + \frac{1}{X}$, Z= $\sqrt{X}$ and W=2X.
36	What is location, scale and shape family of distribution? State its use.
37	What is Marshall-Olkin bivariate distribution? State and prove its forgetfulness property.
38	Suppose that the joint pdf of the two-dimensional random variable (X,Y) is given by,

	$f(x,y) = \begin{cases} x^2 + \frac{xy}{3} & if\ 0 < \text{x} < 1, 0 < \text{y} < 2, \\ 0 & otherwise. \end{cases}$ Compute the following. i. P(X>1/2) ii. P(Y<X) iii. P(Y<0.5|X<0.5)
39	Obtain the moment generating function of a bivariate normal distribution with parameters N$(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$.
40	Let X_1 and X_2 be independent U(0, 1) random variables. Obtain the probability density function of Z = $X_1 + X_2$.
41	Suppose X and Y are iid R.V'S with chi-square distribution with n_1 and n_2 df respectively. Obtain the distribution of $Z = \frac{X}{X+Y}$.
42	A continuous probability distribution has density $f(x) = \begin{cases} a(b-x)^2 & if\ a < x < b, \\ 0 & otherwise. \end{cases}$ If the mean of the distribution is unity, find the values of a and b.
43	If X and Y are jointly distributed with probability density function (p.d.f.) $f(x,y) = \begin{cases} 24\, xy, & if\ x \geq 0, y \geq 0, (x+y) \leq 1. \\ 0, & otherwise \end{cases}$ Find a) Marginal distributions of X and Y. b) Conditional distribution of Y given X = x. c) E (Y / X = x).
44	Show that $f(x,y) = \begin{cases} 4\theta(1 + xy(x^2 - y^2)) & if\ \|x\| < 1, \quad \|y\| < 1 \\ 0 & otherwise \end{cases}$ is a joint density function of a bivariate random variable. Obtain the marginal distribution of X and Y.
45	Let X_1 and X_2 be independent U(0, 1) random variable. Then, find the distribution of Z= X_1 - X_2.
46	Let X and Y are jointly distributed with p.d.f. $f(x,y) = \begin{cases} (x+y), & if\ 0 \leq \text{x} \leq 1, 0 \leq \text{y} \leq 1. \\ 0, & otherwise \end{cases}$ Then, find $P[X > \sqrt{Y}]$.
47	If X_1 ,X_2 ,X_3 ,X_4 are independent N (0,1) random variables. Find the distribution of

	$2(X_1 X_2 + X_3 X_4)$. Also find the distribution of $(\sum X_i)^2$ and check whether these two random variables are independently distributed with quoting all the relevant results required.
48	Let $Y = X_1 + X_2 + \dots\dots\dots + X_N$ where Xi are iid exponential with rate λ and N is a geometric random variable with success probability p independently distributed with Xi. Obtain mean and variance of Y.
49	Let (X, Y, Z) have the joint p.d.f. given by $$f(x,y,z) = \begin{cases} \dfrac{6}{(1+x+y+z)^4} & if\ x,y,z > 0, \\ 0 & otherwise. \end{cases}$$ Find the joint p.d.f. of U = X + Y + Z, V = X + Y, W = X. Hence obtain p.d.f. of U.
50	Derive conditional distribution X on Y of Bivariate Normal Distribution and comment on it.