M.Sc

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Seat No.	

M.Sc. (Part - I) (Semester - II) Examination, 2013

STATISTICS (Paper - VI) (Credit System) (A.F.)

Probability Theory

Sub. Code : 42327

Day and Date : Monday, 15 - 04 - 2013 **Total Marks : 80**

Time : 11.00 a.m. to 2.00 p.m.

Instructions :
1) Question No. 1 is compulsory.
2) Attempt any 4 questions from No.2 to 7.
3) Figures to the right indicate marks to the sub-question.

Q1) Answer any eight of the following : [8 × 2 = 16]

a) Define :
   i) field → finite ii) σ-field → countable
   Give an example of σ-field.

b) Define :
   i) $\lim\sup A_n$ ii) $\lim\inf A_n$
   iii) $\lim A_n$
   where A_n is a sequence of sets.

c) Define :
   i) probability measure ii) generalized probability measure
   Give an example of each.

d) Show that an indicator function is measurable.

e) Define :
   i) simple function ii) random variable

f) State monotone convergence theorem.

g) Define :
   i) Convergence in probability ii) Almost sure convergence

h) Define a characteristic function and state any two properties.

i) State Lindeberg-Feller theorem on CLT.

j) Give an application of CLT.

Q2) a) Let $A_n = A$ if $n = 1, 3, 5, ...$ and $A_n = B$ if $n = 2, 4, 6, ...$
   Obtain $\overline{\lim} A_n$, $\underline{\lim} A_n$. Does $\lim A_n$ exist? Justify your answer. [6]

   No

b) Show that $\underline{\lim} A_n \subseteq \overline{\lim} A_n$. [10]

Q3) a) Define minimal σ-field and give an example of the same. [4]

b) Show that the intersection of arbitrary number of fields is also field. [6]

c) Show that every field contains the empty set ϕ and the whole space Ω. Is a set containing ϕ & Ω always field? Justify your answer. [6]

Q4) a) Define inverse mapping and show that inverse mapping preserves all set relations. [10]

b) If A_1, A_2 are measurable sets and a function X is defined by, [6]

$$X(W)=\begin{cases} -1 & W\in A_1 \\ +1 & W\in A_1^C A_2 \\ 0 & W\in A_1^C A_2^C \end{cases}$$

Examine whether X is measurable.

Q5) a) Show that a random variable X is a finite limit of a sequence of simple random variables. [6]

b) If $A_n \to A$ then show that $P(A_n) \to P(A)$. [6]

c) Define expectation of a random variable. Give an example of a random variable for which expectation does not exist. [4]

Q6) a) Prove: $X_n \xrightarrow{P} 0$ iff $E\dfrac{|X_n|}{1+|X_n|} \to 0$ as $n \to \infty$. [8]

b) Prove: $X_n \xrightarrow{a.s.} X$ iff as $n \to \infty$

$$P\left(\bigcup_{k=n}^{\infty}\left[W : |X_k - X| \geq \tfrac{1}{r}\right]\right) \to 0 \ \forall r, \text{ an integer}$$ [8]

Q7) Write short notes on any four of the following : [4 × 4]

a) Weak and strong law of large numbers.

b) Borel-Cantelli Lemma.

c) Yule Slutsky results.

d) Inversion theorem.

e) Kolmogorov three series criterion.

f) Dominated convergence theorem.

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Seat No.	

M.Sc. (Part - I) (Semester - II) Examination, April - 2014
STATISTICS (Paper -VI) (Revised) (CBCS)
Probability Theory
Sub. Code : 61313

Day and Date : Wednesday, 09- 04 - 2014 **Total Marks : 80**

Time : 11.00 a.m. to 2.00 p.m.

Instructions :
1) Question number 1 is compulsory.
2) Attempt any four questions from question No. 2 to 7.
3) Figures to right indicate marks to the questions.

Q1) Answer the following sub questions. **[16]**

a) Define a field and give an example of a field.

b) Comment on a statement - Every field must contain null set.

c) Define a σ - field.

d) If $A_n = A \ \forall\, n \geq 1$ then what is $\lim A_n$?

e) Define $\lim\inf A_n$ and $\lim\sup A_n$, when A_n is a sequence of sets.

f) Define inverse mapping?

g) What is simple function?

h) What is indicator function?

i) What is an additive property of probability measure?

j) Give an example of a simple random variable.

k) State weak law of large number.

l) Define characteristic function and state a property of it.

m) Define an expectation of a non-negative random variable.

n) State inversion theorem of characteristic function.

o) Define independence of two random variables.

p) State Lindeberg Feller CLT.

Q2) a) Let a sequence of sets $\{A_n\}$ be defined as $A_n = \left[o, 1+\frac{3}{n}\right), n \geq 1$

Obtain $\lim A_n$, if exists. [8]

b) State and prove relationship between lim inf A_n and lim sup A_n. [8]

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Q3) a) Define (i) Probability measure ~~(ii)~~ Conditional probability measure.

Let $\mathcal{A} = \{A, A^c, \Omega, \phi\}$ construct a suitable probability measure on $\mathcal{A}$. [8]

b) If $A_n \uparrow A$ then prove that $P(A_n) \to P(A)$. Comment on similar result for sequence A_n^c. [8]

Q4) a) In usual notations, prove that $\boxed{\sigma\{x^{-1}(\mathcal{C})\} = x^{-1}\{\sigma(\mathcal{C})\}}$ [8]

b) Let $\Omega = \{-2, -1, 0, 1, 2\}$. If $x(i) = i, i = 0, \pm 1, \pm 2$, obtain σ-fields induced by x and x^2. [8]

Q5) a) Define almost sure convergence and convergence in probability.

Let $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$ then prove that $X_n + Y_n \xrightarrow{P} X+Y$. [8]

b) State and prove menotone convergence theorem. [8]

Q6) a) Obtain characteristic function of standard normal variate. [6]

b) State Kolmogerv's three series theorem. [4]

c) Give an application of dominated convergence theorem. [6]

Q7) Write short notes on the following : [4 × 4 =16]

a) Borel - Cantelli Lemna.

b) Convergence in r^{th} mean.

c) Convergence in distribution.

d) Lebesgue - steltjes measure.

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M.Sc. (Part - I) (Semester - II) Examination, April - 2015

STATISTICS (Paper - VI)

Probability Theory (CBCS)

Sub. Code : 61313

Day and Date : Monday, 06- 04 - 2015 **Total Marks : 80**

Time : 10.30 a.m. to 01.30 p.m.

Instructions :
1) Question No. 1 is compulsory.
2) Attempt any four questions from Question No. 2 to 7.
3) Figures to right indicate marks to the questions.

Q1) Answer the following : **[16]**

a) Define an indicator function.

b) Define characteristic function.

c) State any two properties of characteristic function.

d) State Kolmogorov's three series theorem.

e) Define measurable space.

f) Give an example of a measurable space.

g) Give an example of a simple random variable.

h) Define conditional expectation.

i) Define distribution function of a random variable.

j) State Yule-Slutsky result.

k) State Lindberg-Feller Theorem on CLT.

l) Comment on the statement: Every field must contain ϕ and Ω.

m) Comment on the statement: Indicator function is a random variable.

n) State weak law of large numbers.

o) Define Borel field.

p) Define counting measure.

Q2) a) Define a Monotone field. Show that σ-field is a monotone field. Is converse true ? Justify. **[8]**

b) Define inverse mapping and show that inverse mapping preserves all set relations. **[8]**

Q3) a) Define **[8]**

i) Lebesgue-Steiltje's Measure

ii) Probability measure

iii) Field

iv) σ- field

b) Define random variable. Is Borel function of a random variable also a random variable ? Justify. **[8]**

Q4) a) Find lim A_n if exists for the following sequence of sets : **[8]**

i) $A_n = \begin{cases} A & \text{if } n \text{ is even} \\ B & \text{if } n \text{ is odd} \end{cases}$

ii) $A_n = (1+\frac{1}{n}, 2+\frac{1}{n})$

b) Discuss **[8]**

i) Bernoulli random variable

ii) Poisson random variable

Q5) a) State and prove dominated convergence theorem. **[8]**

b) Define convergence in distribution and convergence in probability. Show that convergence in probability implies convergence in distribution. Is converse true ? Justify. **[8]**

Q6) a) State and prove Borel-Cantelli lemma. **[6]**

b) Explain the terms with illustrations. **[6]**

i) Mutual Independence.

ii) Pairwise Independence.

c) Give an example of a mapping which is not a random variable. Justify. **[4]**

Q7) Write short notes on the following : **[4 × 4 = 16]**

a) Fatou's lemma.

b) Almost sure convergence.

c) Continuity property of probability measure.

d) Probability measure induced by random variable X.

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M.Sc. (Part - I) (Semester - II) (C.B.C.S.)
Examination, April - 2016
STATISTICS
Probability Theory (Paper - VI)
Sub. Code : 61313

Day and Date : Friday, 01 - 04 - 2016 **Total Marks : 80**

Time : 11.00 a.m. to 2.00 p.m.

Instructions :
1) Question No. 1 is compulsory.
2) Attempt any four questions from Question No. 2 to 7.
3) Figures to right indicate marks to the questions.

Q1) Attempt the following: **[16 × 1 = 16]**

a) Define Borel σ-field.

b) Define limit of monotone increasing sequence of sets.

c) Examine a class of finite and cofinite sets to be a field.

d) Give an example of a field which is not monotone.

e) Let $\Omega = \mathbb{R}$ for $X(W) = |W|$, write down $X^{-1}(B)$ for $B \in \mathbb{R}$.

$A \cup A^c = \Omega$

f) Prove or disprove : $I(A^C) = 1 - I(A)$.

g) Give an example of a mapping which is not a random variable.

h) In usual notations, show that $P(\phi) = 0$.

$A \cap A^c = \phi$

i) Write down the Lebesgue measure of set A, where A is set of all rationals in (0, 1).

j) Define Lebesgue - steiltje's measure.

P.T.O.

k) State Liaponov's from of CLT.

l) State Borel-Cantelli Lemma.

m) Prove on disprove: If $X \geq 0$ a.s. then $E\,X \geq 0$.

n) Define pairwise independence.

o) State Kolmogorov's three series theorem.

p) In usual notations, show that $|\phi(t)| \leq 1$.

$\phi(t) \leq 1$

$\phi(0) = 1$

Q2) a) Explain limit of arbitrary sequence of sets. Obtain limit of the sequence of sets defined by

$$\left.\begin{array}{l} A_{2n} = \left(0, \dfrac{1}{2n}\right) \\ A_{2n+1} = \left[-1, \dfrac{1}{2n+1}\right] \end{array}\right\} n = 1, 2, \ldots.$$

if exists.

b) Define σ-field with example.

Prove or disprove : An arbitrary intersection of σ-fields is also a σ-field.

[8 + 8]

Q3) a) Show that inverse mapping preserves set relations.

b) Explain probability measure induced by a random variable X. Obtain Binomial random variable through usual probability measure P and corresponding induced probability measure.

[8 + 8]

Q. a) Explain expectation of an arbitrary random variable. Show that X is integrable iff $|X|$ is integrable.

b) State and prove monotone convergence theorem.

[8 + 8]

Q. a) Define Borel function. Show that Borel function of a random variable is also a random variable.

b) Explain Lindberg-Felier theorems on CLT. Give one application of the same.

[8 + 8]

Q. a) Prove or disprove : $X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{D} X$.

b) Prove or disprove : $X_n \xrightarrow{a.s.} X \Rightarrow X_n \xrightarrow{r} X$.

c) Obtain characteristics function of $N(\mu, \sigma^2)$.

d) Let B_n be a non-decreasing sequence of sets belonging to $\mathcal{F}$ and $(\Omega \mathcal{F}, p)$ be the probability space. Then show that

$$P(B_n) \uparrow P(B), \text{ where, } B = \bigcup_{n=1}^{\infty} B_n.$$

[4 × 4]

Q. Write short notes on the following: [4 × 4]

a) Dominated convergence theorem.

b) Characteristic function and its properties.

c) Indicator function and its properties.

d) Economical difinition of a random variable.

Seat No.

M.Sc. (Part - I) (Semester - II) (Credit System) Examination, November - 2014

STATISTICS (Paper - VI)

Probability Theory (A.F)

Sub. Code : 42327

Day and Date : Monday, 10 - 11 - 2014 **Total Marks : 80**

Time : 02.30 p.m. to 5.30 p.m.

Instructions :
1) Question No. 1 is compulsory.
2) Attempt any 4 questions from No. 2 to 7.
3) Figures to right indicate marks.

Q1) Answer any eight of the following : [8 × 2 = 16]

a) Define field and σ - field. Give an example of a field which is not a σ - field.

b) Show that the limit of a monotone sequence of sets exists.

c) If $\{A_n\}$ is a sequence of events on probability space (Ω, A, P), Show that $P\left(\bigcap_{n=1}^{b} A_n\right) = 1$, when $P(A_n) = 1, \forall n$.

d) Define Lebesgve - steiltije (LS) measure and state relationship between LS measure and probability measure.

e) Define Lim inf and lim sup of a sequence of sets $\{An\}$. Find $\lim A_n$ if $A_n = \left(2 - \frac{1}{n}, 3 + \frac{1}{n}\right)$.

f) Define convergence in distribution of a sequence of random variables.

g) Define a monotone field.

h) State Kolmogorov zero - one law. Give its application.

i) Let A, B, C be three events. Define pairmise independence and mutual independence of these events.

j) State Khintchine's WLLN.

Q2) a) Prove that the intersection of arbitrary number of fields is a field. **[8]**

b) Examine whether the sequence. $A_n = \left\{ w : 0 < w < b + \frac{(-1)^n}{n} \right\}$ is convergent or not, given $b > 1$. **[8]**

Q3) a) Define a probability space (Ω, A, P), if $\{A_n\}$ is sequence of events such that lim A_n exists, prove that $P\left(\lim_{n\to\infty} A_n\right) = \lim_{n\to\infty} P(A_n)$. **[8]**

b) Define a random variable. If $x_1, \ldots, x_n$ are random variables, show that $x_1 + x_2$, $max\, x_j$ $1 \le j \le n$ are also random variables. **[8]**

Q4) a) Define Characteristic function of a random variable obtain the same for normal distribution. **[8]**

b) Using inversion formula, obtain the distribution whose characteristic function is $Q(t) = e^{-|t|}$. **[8]**

Q5) a) State and prove Borell - cantelli lemma. **[12]**

b) Show that if $|X_n| \le y$, y integrable, then

$X_n \xrightarrow{a.s} X \Rightarrow E(X_n) \longrightarrow E(X)$ **[4]**

Q6) a) State Lindberg - feller and Liapunov form of central limit theorem. (CLT) Give an application of CLT. **[8]**

b) Establish relationship between conversence in probability and almost sure convergence. **[8]**

Q7) Write short notes on any four **[4 × 4]**

a) Borel field

b) Probability measure and countable measure.

c) Limits of sequence of random variables

d) Dominated convergence theorem.

e) Yule - Shitsky results.

f) Convergence of sequence in distribution.

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Total No. of Pages : 2

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M.Sc. (Part - I) (Semester - II) Examination, 2013

STATISTICS (Paper - VI) (Credit System) (A.F.)

Probability Theory

Sub. Code : 42327

Day and Date : Monday, 15 - 04 - 2013 **Total Marks : 80**

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   Give an example of σ-field.

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   iii) $\lim A_n$
   where A_n is a sequence of sets.

c) Define :
   i) probability measure ii) generalized probability measure
   Give an example of each.

d) Show that an indicator function is measurable.

e) Define :
   i) simple function ii) random variable

f) State monotone convergence theorem.

g) Define :
   i) Convergence in probability ii) Almost sure convergence

h) Define a characteristic function and state any two properties.

i) State Lindeberg-Feller theorem on CLT.

j) Give an application of CLT.

Q2) a) Let $A_n = A$ if $n = 1, 3, 5, \ldots$ and $A_n = B$ if $n = 2, 4, 6, \ldots$

Obtain $\overline{\lim}\, A_n$, $\underline{\lim}\, A_n$. Does $\lim A_n$ exist? Justify your answer. [6]

NO

b) Show that $\underline{\lim}\, A_n \subseteq \overline{\lim}\, A_n$. [10]

Q3) a) Define minimal σ-field and give an example of the same. [4]

b) Show that the intersection of arbitrary number of fields is also field. [6]

c) Show that every field contains the empty set ϕ and the whole space Ω. Is a set containing ϕ & Ω always field? Justify your answer. [6]

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b) If A_1, A_2 are measurable sets and a function X is defined by, [6]

$$X(W)=\begin{cases}-1 & W\in A_1\\ +1 & W\in A_1^C A_2\\ 0 & W\in A_1^C A_2^C\end{cases}$$

Examine whether X is measurable.

Q5) a) Show that a random variable X is a finite limit of a sequence of simple random variables. [6]

b) If $A_n \to A$ then show that $P(A_n)\to P(A)$. [6]

c) Define expectation of a random variable. Give an example of a random variable for which expectation does not exist. [4]

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b) Prove: $X_n \xrightarrow{a.s.} X$ iff as $n\to\infty$

$$P\left(\bigcup_{k=n}^{\infty}\left[W:|X_k - X|\geq \tfrac{1}{r}\right]\right)\to 0 \ \forall r, \text{ an integer}$$ [8]

Q7) Write short notes on any four of the following : [4 × 4]

a) Weak and strong law of large numbers.

b) Borel-Cantelli Lemma.

c) Yule Slutsky results.

d) Inversion theorem.

e) Kolmogorov three series criterion.

f) Dominated convergence theorem.

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