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M.Sc. (Part – II) (Semester – IV) (CGPA) Examination, 2016
STATISTICS (Paper – XVIII)
Reliability and Survival Analysis (New)

Day and Date : Monday, 4-4-2016

Total Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

- Instructions:** 1) Attempt **five** questions.
2) Q. No. 1 and Q. No. 2 are **compulsory**.
3) Attempt **any three** from Q. No. 3 to Q. No. 7.
4) Figures to the **right** indicate **full** marks.

1. A) Choose the correct alternative :

- 1) The total time on test corresponding to a complete sample is
 - a) $x_{(n)}$
 - b) $nx_{(n)}$
 - c) $n\bar{x}$
 - d) none of these
- 2) Log-rank test for equality of two distributions is based on _____ data.
 - a) type I censoring
 - b) type II censoring
 - c) left censored
 - d) right censored
- 3) For a parallel system of two independent components having reliability 0.6 each, then the reliability of system is
 - a) 0.4
 - b) 0.6
 - c) 0.84
 - d) 0.16
- 4) Parallel system of n components has _____ minimal path set.
 - a) 1
 - b) n
 - c) 2^n
 - d) 2^{n-1}
- 5) Which of the following is coherent system ?
 - a) parallel
 - b) series
 - c) k-out-of-n
 - d) all the above



B) Fill in the blanks :

- 1) DFRA class is preserved under _____
- 2) A function is star shaped if _____
- 3) K-M estimator of survival function $S(t)$ when there are no ties in an experiment is given by _____
- 4) The number of minimal paths in 2 out of 3 system is _____
- 5) The i^{th} component is said to be irrelevant to structure function ϕ if _____

C) State whether the following statements are **true** or **false** :

- 1) The dual of K out of n system is (n-k) out of n system.
- 2) The subset of minimal cut set is cut set.
- 3) Type I censoring is a particular case of random censoring.
- 4) Kaplan-Meier estimator is nonparametric in nature.

(5+5+4)

2. a) Define :

- i) Mean Time To Failure (MTTF).
- ii) Mean Residual Life (MRL) function.
- iii) Failure rate function.

b) Write short note on the following :

- i) Gehan's test.
- ii) NBU and MBUE class of life distributions.

(6+8)

3. a) Define coherent structure for a coherent structure with n components.

Prove that $\prod_{i=1}^n x_i \leq \phi(\underline{x}) \leq \prod_{i=1}^n x_i$.

b) If x_1, x_2, \dots, x_n are associated binary random variables, then prove that

$$P\left[\prod_{j=1}^n x_j = 1\right] \geq \prod_{j=1}^n P(x_j = 1).$$

(7+7)



4. a) Define Polya function of order 2 (PF_2). If $f \in PF_2$, then prove that $F \in IFR$.
b) Define IFR and IFRA class of distributions. If $F \in IFR$ then prove that $F \in IFRA$. **(7+7)**
5. a) Describe each of the following with one simple illustration.
i) Type I censoring.
ii) Type II censoring.
iii) Random censoring.
b) Describe actuarial method of estimation of survival function. **(7+7)**
6. a) Describe Deshpande's test for exponentiality against IFRA.
b) Discuss maximum likelihood estimation of parameters of gamma distribution based on complete sample. **(7+7)**
7. a) Explain Mantel's technique of computing Gehan's statistic for a two sample problem for testing equality of two life distributions.
b) Obtain MLE of mean of exponential distribution under type II censoring. **(7+7)**
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- 11) For which of the following family, each member has non-monotonic failure rate?
 a) exponential b) log-normal
 c) Weibull d) Gamma

- 12) Let $X_{(r)}$ be the r^{th} order statistic in a random sample of size n taken from exponential distribution with mean θ . Then $E[X_{(r)}] = \underline{\hspace{2cm}}$.
 a) $\theta[n - r + 1]^{-1}$ b) $\sum_{i=1}^r [n - i + 1]^{-1}$
 c) $\sum_{i=1}^r \theta [n - i + 1]^{-1}$ d) $\frac{1}{\theta} \sum_{i=1}^r [n - i + 1]$

- 13) Which one of the following is not true?
 a) When there is no censoring K-M estimator is empirical distribution function
 b) K-M estimator always exists
 c) K-M estimator is self consistent
 d) K-M estimator is also known as product moment estimator

- 14) The censoring time for every censored observation is identical in _____ censoring.
 a) type I b) type II
 c) random d) both in a and b

Q.2 A) Answer the following questions. (Any Four) 08

- 1) Define IFR and IFRA class of distributions.
- 2) Define associated random variables and state any two properties of associated random variables.
- 3) Give two definitions of star shaped function.
- 4) Describe random censoring with suitable example
- 5) Define Kaplan-Meier estimator.

B) Write notes. (Any Two) 06

- 1) Getian's two sampling test under censoring
- 2) Burnham's measure of structural importance
- 3) Star shaped function

Q.3 A) Answer the following questions. (Any Two) 08

- 1) Define minimal path set and minimal cut set.
- 2) Show that if F is IFR then F is IFRA
- 3) Discuss maximum likelihood estimation of parameters of Weibull distribution based on a complete sample.

B) Answer the following questions. (Any One) 06

- 1) Obtain MLE for mean of exponential distribution under type II censoring.
- 2) For a coherent system with n components, prove that:
 - i) $\phi(0)=0$ and $\phi(1)=1$
 - ii) $\prod_{i=1}^n x_i \leq \phi(x) \leq \prod_{i=1}^n x_i$

Q.4 A) Answer the following questions. (Any Two) 10

- 1) Define mean residual life function and obtain the same for exponential distribution.
- 2) Obtain the likelihood function under random censoring setup, when the observations come from a distribution F with density F .

3) Give two real life examples where both left and right censoring occurs.

B) Answer the following questions. (Any One) 04

1) Describe Kaplan-Meier estimator and derive an expression for the same.

2) Define K-out-of-n system. Obtain the reliability function of the system.

Q.5 Answer the following questions. (Any Two) 14

a) Explain Mantel's technique of computing Gehan's statistic for a two-sample problem for testing equality of two life distributions.

b) Define mean time to failure (MTTF) and mean residual life (MRL) function. Obtain the same for exponential distribution.

c) Show that IFR class of life distributions is closed under convolution.



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M.Sc. (Part – II) (Semester – IV) Examination, 2015
STATISTICS (Paper – XVIII)
Reliability and Survival Analysis

Day and Date : Tuesday, 21-4-2015

Total Marks : 70

Time : 3.00 p.m. to 6.00 p.m.

- Instructions :** 1) Attempt **five** questions.
2) Q. No. (1) and Q. No. (2) are **compulsory**.
3) Attempt **any three** from Q. No. (3) to Q. No. (7).
4) Figures to the **right** indicate **full** marks.

1. A) Choose the correct alternative :

5

1) For which of the following family each member has non-monotone failure rate ?

- a) Exponential
b) Weibull
c) Gamma
d) Lognormal

2) For Weibull distribution _____ parameter decides whether distribution belongs to the IFR class or the DFR class.

- a) Location
b) Shape
c) Scale
d) All the above

3) In censored data actuarial estimator of survival function is _____

- a) $\sum_{i=1}^k \hat{P}_i$
b) $\frac{1}{k} \sum_{i=1}^k \hat{P}_i$
c) $\prod_{i=1}^k \hat{p}_i$
d) $\prod_{i=1}^k (\hat{p}_i)^{\frac{1}{2}}$

4) The i^{th} component of a system is relevant if _____

- a) $\phi(1_i, \underline{x}) = 1$ and $\phi(0_i, \underline{x}) = 1$
b) $\phi(1_i, \underline{x}) = 1$ and $\phi(0_i, \underline{x}) = 0$
c) $\phi(1_i, \underline{x}) = 0$ and $\phi(0_i, \underline{x}) = 0$
d) $\phi(1_i, \underline{x}) = 0$ and $\phi(0_i, \underline{x}) = 1$



5) A function $g(x)$ defined on $[0, \infty)$ is star shaped function if for $0 \leq \alpha \leq 1$,

a) $g(\alpha x) \leq [g(x)]^\alpha$

b) $g(\alpha x) \geq [g(x)]^\alpha$

c) $g(\alpha x) \leq \alpha g(x)$

d) $g(\alpha x) \geq \alpha g(x)$

B) Fill in the blanks :

5

1) IFR property is preserved under _____

2) $F \in$ IFRA if and only if $-\log R(t)$ is _____

3) A sequence of (2×2) contingency tables is used in _____

4) In type II censoring _____ is fixed.

5) The minimal path sets for a structure ϕ are _____ for its dual.

C) State whether the following statements are **True** or **False** :

4

1) The dual of a parallel system is not a parallel.

2) For exponential distribution failure rate is constant.

3) Weibull distribution is IFR for all parameter values.

4) IFRA property is preserved under convolution.

2. a) Answer the following :

6

1) Define minimal path sets and minimal cut sets. Illustrate the same by example.

2) Define k out of n system. Obtain the reliability function of this system.

b) Write short notes on the following :

8

1) Empirical survival function and its properties.

2) Log-rank test.

3. a) Define NBU and NBUE classes of distributions. Prove that

$$F \in \text{IFRA} \Rightarrow F \in \text{NBU}.$$

b) Give two definitions of star shaped function and prove their equivalence. (7+7)

4. a) Describe various censoring schemes.

b) Obtain MLE of the mean (θ) of an exponential distribution based on type I and type II censoring. (7+7)



5. a) Describe situations where random censoring occurs naturally. Obtain actuarial estimate of survival function and derive Greenwood's formula for the estimate of variance of the estimator.
- b) Define associated random variables. If X_1, X_2, \dots, X_n are binary associated random variables then prove that $P \left[\prod_{j=1}^n X_j = 1 \right] \leq \prod_{j=1}^n P(X_j = 1)$. (7+7)
6. a) Let $F(x) = \int F_\alpha(x) dG(\alpha)$ be a mixture of $\{F_\alpha\}$ with mixing distribution $G(\alpha)$. Prove that if each F_α is DFR then F is DFR.
- b) Obtain the actuarial estimator of the survival function. Clearly state the assumption that you need to make. State Greenwood's formula for the variance of the estimator. (7+7)
7. a) Show that Kaplan-Meier estimator of survival function is the generalized likelihood estimator of the survival function.
- b) Define structure function of a system. Obtain structure function of a system in terms of minimal path sets. (7+7)
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**M.Sc. (Part – II) (Semester – IV) Examination, 2015
STATISTICS (Paper – XVIII)
Reliability and Survival Analysis**

Day and Date : Tuesday, 21-4-2015

Total Marks : 70

Time : 3.00 p.m. to 6.00 p.m.

- Instructions :** 1) Attempt *five* questions.
2) Q. No. (1) and Q. No. (2) are **compulsory**.
3) Attempt **any three** from Q. No. (3) to Q. No. (7).
4) Figures to the **right** indicate **full marks**.

1. A) Choose the correct alternative : 5

- 1) For which of the following family each member has non-monotone failure rate ?
- a) Exponential
b) Weibull
c) Gamma
d) Lognormal
- 2) For Weibull distribution _____ parameter decides whether distribution belongs to the IFR class or the DFR class.
- a) Location
b) Shape
c) Scale
d) All the above
- 3) In censored data actuarial estimator of survival function is _____
- a) $\sum_{i=1}^k \hat{P}_i$
b) $\frac{1}{k} \sum_{i=1}^k \hat{P}_i$
c) $\prod_{i=1}^k \hat{p}_i$
d) $\prod_{i=1}^k (\hat{p}_i)^{\frac{1}{2}}$
- 4) The i^{th} component of a system is relevant if _____
- a) $\phi(1, \underline{x}) = 1$ and $\phi(0, \underline{x}) = 1$
b) $\phi(1, \underline{x}) = 1$ and $\phi(0, \underline{x}) = 0$
c) $\phi(1, \underline{x}) = 0$ and $\phi(0, \underline{x}) = 0$
d) $\phi(1, \underline{x}) = 0$ and $\phi(0, \underline{x}) = 1$



5) A function $g(x)$ defined on $[0, \infty)$ is star shaped function if for $0 \leq \alpha \leq 1$,

a) $g(\alpha x) \leq [g(x)]^\alpha$

b) $g(\alpha x) \geq [g(x)]^\alpha$

c) $g(\alpha x) \leq \alpha g(x)$

d) $g(\alpha x) \geq \alpha g(x)$

B) Fill in the blanks :

5

1) IFR property is preserved under _____

2) $F \in \text{IFRA}$ if and only if $-\log R(t)$ is _____

3) A sequence of (2×2) contingency tables is used in _____

4) In type II censoring _____ is fixed.

5) The minimal path sets for a structure ϕ are _____ for its dual.

C) State whether the following statements are **True** or **False** :

4

1) The dual of a parallel system is not a parallel.

2) For exponential distribution failure rate is constant.

3) Weibull distribution is IFR for all parameter values.

4) IFRA property is preserved under convolution.

2. a) Answer the following :

6

1) Define minimal path sets and minimal cut sets. Illustrate the same by example.

2) Define k out of n system. Obtain the reliability function of this system.

b) Write short notes on the following :

8

1) Empirical survival function and its properties.

2) Log-rank test.

3. a) Define NBU and NBUE classes of distributions. Prove that

$$F \in \text{IFRA} \Rightarrow F \in \text{NBU}.$$

b) Give two definitions of star shaped function and prove their equivalence. (7+7)

4. a) Describe various censoring schemes.

b) Obtain MLE of the mean (θ) of an exponential distribution based on type I and type II censoring. (7+7)



5. a) Describe situations where random censoring occurs naturally. Obtain actuarial estimate of survival function and derive Greenwood's formula for the estimate of variance of the estimator.
- b) Define associated random variables. If X_1, X_2, \dots, X_n are binary associated random variables then prove that $P \left[\prod_{j=1}^n X_j = 1 \right] \leq \prod_{j=1}^n P(X_j = 1)$. (7+7)
6. a) Let $F(x) = \int F_\alpha(x) dG(\alpha)$ be a mixture of $\{F_\alpha\}$ with mixing distribution $G(\alpha)$. Prove that if each F_α is DFR then F is DFR.
- b) Obtain the actuarial estimator of the survival function. Clearly state the assumption that you need to make. State Greenwood's formula for the variance of the estimator. (7+7)
7. a) Show that Kaplan-Meier estimator of survival function is the generalized likelihood estimator of the survival function.
- b) Define structure function of a system. Obtain structure function of a system in terms of minimal path sets. (7+7)
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M.Sc. (Part – II) (Semester – IV) Examination, 2014
STATISTICS (Paper – XVIII)
Reliability and Survival Analysis

Day and Date: Saturday, 26-4-2014

Total Marks : 70

Time: 3.00 p.m. to 6.00 p.m.

- Instructions :** 1) Attempt **any five** questions.
2) Q. No. (1) and Q. No. (2) are **compulsory**.
3) Attempt **any three** from Q. No. (3) to Q. No. (7).
4) Figures to the **right** indicate **full** marks.

1. A) Choose the correct alternative : 5
- 1) In type II censoring
- duration of experiment is fixed
 - number of failures is fixed
 - both time and number of failures are fixed
 - none of these
- 2) For which of the following family, each member has non-monotonic failure rate ?
- exponential
 - weibull
 - lognormal
 - gamma
- 3) IFRA property is preserved under
- coherent
 - mixture
 - convolution
 - none of these
- 4) Birnbaum's measure of structural importance of component i is
- $\frac{n_{\phi(i)}}{n^2}$
 - $\frac{n_{\phi(i)}}{(n-1)^2}$
 - $\frac{n_{\phi(i)}}{2^{n-1}}$
 - $\frac{n_{\phi(i)}}{2^n}$
- 5) The i^{th} component of a system is relevant if
- $\phi(1, \underline{x}) = 1$
 - $\phi(0, \underline{x}) = 0$
 - both (a) and (b)
 - neither (a) nor (b)



- B) Fill in the blanks : 5
- 1) If p_i is the reliability of the i^{th} component, then the reliability of parallel system of n independent components is _____
 - 2) A function is star shaped if _____
 - 3) A distribution F is said to be NBU if _____
 - 4) Mantel derived variance of Gehan's statistic under the hypothesis _____
 - 5) Kaplan-Meier estimator is also called _____ estimator.
- C) State whether the following statements are **True** or **False** : 4
- 1) Weibull distribution has monotone failure rate.
 - 2) Non-decreasing function of associated random variables is not associated.
 - 3) Log-rank test is based in left censored data.
 - 4) Type I censoring is a particular case of random censoring.
2. a) Answer the following : 6
- 1) Define dual of structure function. Obtain dual of 2-out-of-3 system.
 - 2) Describe type I censoring.
- b) Write short notes on the following : 8
- 1) Star shaped function.
 - 2) Log-rank test.
3. a) If $h(p)$ is the reliability function of coherent system of n independent components then show that $h(p^\alpha) \geq [h(p)]^\alpha$.
- b) Define IFR and IFRA class of distributions. If $F \in \text{IFR}$ then show that $F \in \text{IFRA}$. (6+8)
4. a) Define (i) NBU (ii) NBUE class of distributions. Prove or disprove : $\text{NBU} \Rightarrow \text{NBUE}$.
- b) If failure time of item has the Pareto distribution. Obtain failure rate function and show that the distribution belongs to DFR.
- c) Define reliability function. Obtain it for series and parallel system of n independent components. (5+4+5)



5. a) Discuss maximum likelihood estimation of parameters of weibull distribution based on a complete sample.
 - b) Define actuarial estimator and obtain an estimate of variance of actuarial estimator.
 6. a) Describe each of the following with one simple illustration :
 - i) Type I censoring
 - ii) Type II censoring
 - iii) Random censoring.
 - b) Describe two sample problem under randomly censored set up and develop Gehan's test for the same. **(6+8)**
 7. a) Obtain m.l.e. for mean of the exponential distribution under Type II censoring.
 - b) Describe the Kaplan-Meier estimator.
 - c) Define mean residual life function and obtain the same for exponential distribution. **(5+5+4)**
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Seat
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**M.Sc. (Semester – IV) (New) (CBCS) Examination March/April-2019
Statistics**

RELIABILITY AND SURVIVAL ANALYSIS

Day & Date: Wednesday, 24-04-2019
Time: 03:30 PM To 06:00 PM

Max. Marks: 70

Instructions: 1) All questions are compulsory.
2) Figures to the right side indicate full marks.

Q.1 Choose the correct alternative:**14**

- 1) Which of the following is coherent system?
 - a) Series
 - b) Parallel
 - c) K out of n
 - d) All the above
- 2) IFR class is preserved under _____.
 - a) coherent
 - b) convection
 - c) mixture
 - d) all the above
- 3) A function $g(x)$ defined on $[0, \infty]$ is star shaped if _____.
 - a) $\frac{g(x)}{x} \uparrow x$
 - b) $\frac{g(x)}{x} \downarrow x$
 - c) $g(\alpha x) \geq \alpha g(x)$
 - d) $g(\alpha x) = \alpha g(x)$
- 4) If P_i is the reliability of i^{th} component then reliability of series system of n independent components is _____.
 - a) $\sum_{i=1}^n P_i$
 - b) $\prod_{i=1}^n P_i$
 - c) $\prod_{i=1}^n (1 - P_i)$
 - d) $1 - \prod_{i=1}^n (1 - P_i)$
- 5) Which of the following rate function corresponds in DFR distribution?
 - a) $h(t) = t$
 - b) $h(t) = te^t$
 - c) $h(t) = e^{-t}$
 - d) None of these
- 6) In a system of n components, total number of path and cut vectors is _____.
 - a) n
 - b) 2^n
 - c) 2^{n-1}
 - d) ${}^n C_2$
- 7) In a K out of n system there are _____ minimal path sets.
 - a) n
 - b) k
 - c) ${}^n C_k$
 - d) n^k
- 8) An experiment is terminated after a fixed time under _____ censoring scheme.
 - a) Type I
 - b) Type II
 - c) Both (a) and (b)
 - d) Neither (a) nor (b)
- 9) Logrank test for equality of two distributions is based on _____ data.
 - a) Type I censoring
 - b) Type II censoring
 - c) Left censoring
 - d) Right censoring
- 10) The TTT transforms of an IFR distribution is _____.
 - a) Convex
 - b) Concave
 - c) Linear
 - d) Neither concave nor convex

B) Attempt any one of the following : **04**

- 1) Derive the likelihood function of observed data under type I censoring.
- 2) Obtain the structure function of a coherent system using minimal path sets. Illustrate the same by an example.

Q.5 Answer any two of the following : **14**

- a) Define NBU and NBUE class of distributions.
Prove or disprove : $NBU \Rightarrow NBUE$
- b) Define associated random variables. If X_1, X_2, \dots, X_n are binary associated random variables then prove that $1-X_1, 1-X_2, \dots, 1-X_n$ are associated.
- c) Define TTT transform. Show that for an IFR distribution TTT transform is a convex function