SL - 270

Total No. of Pages : 2

Seat No.	

M.Sc. (Part - I) (Semester - I) (NEP)
Examination, December - 2023
STATISTICS/APPLIED STATISTICS AND INFORMATICS
Estimation Theory (Paper - IV)
Sub. Code : 87866/87780

Day and Date : Monday, 04 - 12 - 2023

Total Marks : 80

Time : 10.30 a.m. to 1.30 p.m.

Instructions :
1) Question number 1 is compulsory.
2) Attempt any four questions from question numbers 2 to 7.
3) Figures to the right indicate full marks.

Q1) Answer the following. [8×2=16]

a) Define sufficient statistic. Obtain a sufficient statistic for the variance of an exponential distribution based on a sample of size n.

b) Define bounded completeness. Give an example of a bounded compete family.

c) State Rao - Blackwell theorem.

d) If x follows $P(\lambda)$, obtain Fisher information of λ^2.

e) State the invariance property of MLE.

f) Define kernel and U-statistic for an estimable parameter.

g) Define prior and posterior distributions.

h) Define conjugate family of distributions. Give an example.

Q2) a) Define one parameter exponential family of distributions. Obtain minimal sufficient statistic for this family. [8+8]

b) Define ancillary statistic. State and prove Basu's theorem.

Q3) a) State and prove Lehman - Scheffe theorem. [8+8]

b) Define UMVUE. Suppose $X_1, X_2, \ldots\ldots\ldots, X_n$ are iid random variable from Poisson (θ) distribution. Obtain UMVUE of $e^{-\theta}$.

P.T.O.

Q4) a) Describe method of scoring to obtain MLE with suitable example. **[8+8]**

b) Describe method of moments. Let $X_1, X_2, \ldots\ldots\ldots, X_n$ be a sample from $G(\alpha,\beta)$ distribution. Obtain the method of moment estimator of (α,β).

Q5) a) Describe squared error loss functions and absolute error loss functions. Obtain Bayes estimator under squared error loss. **[8+8]**

b) Let $X_1, X_2, \ldots\ldots\ldots, X_n$ be a random sample from $N((\theta,1))$, $\theta \in R$, distribution. The prior distribution of θ is N(0,1). Find the Bayes estimator of θ under squared error loss function.

Q6) a) Let X_1, X_2, X_3 be a random sample from $b(1,\theta)$. Show that X_1+2X_2 is not sufficient statistic for θ. **[4×4]**

b) Let X ~ Geometric (p) distribution. Obtain UMVUE of p based on a random sample of size n.

c) Suppose $X_1, X_2, \ldots\ldots\ldots, X_n$ is a random sample from $f_\theta(x) = e^{-(x-\theta)}, x \geq \theta$. Obtain MLE of θ. Is it unbiased?

d) Explain with examples:

i) non informative prior

ii) Jeffrey's prior.

Q7) Write short notes on the following. **[4×4]**

a) Minimal sufficient partition

b) Crammer - Rao inequality

c) Minimum chi-square estimation

d) Bayes and minimax rules.

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Total No. of Pages : 3

Seat No.	

M.Sc. (Part - I) (Semester - I) (NEP 2020) Examination, December - 2023
STATISTICS/ APPLIED STATISTICS AND INFORMATICS (Paper - II)
Estimation Theory
Sub. Code : 92499

Day and Date : Monday, 04 - 12 - 2023 **Total Marks : 80**

Time : 10.30 a.m. to 1.30 p.m.

Instructions :
1) Question No.1 is compulsory..
2) Attempt any four questions from question numbers 2 to 7.
3) Figures to right indicate full marks.

Q1) Answer the following. **[8×2=16]**

a) State Basu's theorem.

b) Define curved exponential family. Give an example.

c) What is the Fisher's information contained in a single observation drawn from $N(\mu,1)$ distribution?

d) State Cramer-Rao inequality.

e) Define degree and kernel of an estimable parameter.

f) State the invariance property of MLE.

g) Define CAN and BAN estimators.

h) Is consistent estimator unique? Justify.

P.T.O.

Q2) a) Define completeness and bounded completeness of a family of distributions. If $X_1, X_2, \ldots X_n$, is a random sample from $N(\theta, \theta^2)$, then show that $T = \left(\sum X_i, \sum X_i^2\right)$ is sufficient but not complete statistic.

b) State Neyman's factorization theorem and prove it in case of discrete family of distributions.

[8+8]

Q3) a) State and prove a necessary and sufficient condition for an estimator of a parametric function $\psi(\theta)$ to be UMVUE.

b) Define UMVUE. Suppose $X_1, X_2, \ldots, X_n$ are iid random variables from Poisson(θ) distribution. Obtain UMVUE of $e^{-\theta}$.

[8+8]

Q4) a) Suppose $X_1, X_2, \ldots, X_n$, is a random sample from $f_\theta(x) = e^{-(x-\theta)}$, $x \geq \theta, = 0$, *otherwise*. Obtain MLE of θ. Check whether it is unbiased.

b) Describe method of moments estimation. Let $X_1, X_2, \ldots, X_n$ be a sample from $G(\alpha, \beta)$ distribution. Obtain the method of moment estimator of (α, β).

[8+8]

Q5) a) Define weak consistency and strong consistency. Let $X_1, X_2, \ldots, X_n$ be a random sample from exponential distribution with mean θ, show that $\overline{X}$ is consistent for θ whereas $nX_{(1)}$ is not consistent for θ.

b) Let $X_1, X_2, \ldots, X_n$, be iid random variables with PDF $f(x, \theta) = \theta x^{(\theta-1)}, 0 < x < 1, \theta > 0$. Obtain a CAN estimator for θ.

[8+8]

Q6) a) Define one parameter exponential family of distributions. Obtain minimal sufficient statistic for this family.

b) Define unbiased estimator and state its properties.

c) Describe method of scoring.

d) Let $X \sim b(1, p)$. Obtain a CAN estimator of $p(1 - p)$.

[4×4]

Q7) Write short notes on the following. [4×4]

a) Pitman family of distributions.

b) Chapman-Robbins-Kiefer bound.

c) U-statistics.

d) Methods of obtaining CAN estimators.

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Estimation Theory

Total No. of Pages : 2

Seat No.	

M.Sc. (Part - I) (Semester - I) (CBCS) Examination, November - 2019

STATISTICS/APPLIED STATISTICS AND INFORMATICS

Estimation Theory (Paper - IV) (Revised)

Sub. Code : 74910/74977

Day and Date : Friday, 29 - 11 - 2019 **Total Marks : 80**

Time : 11.00 a.m. to 02.00 p.m.

Instructions :
1) **Question No. 1 is compulsory.**
2) **Attempt any four questions from question numbers 2 to 7.**
3) **Figures to the right indicate full marks.**

Q1) Answer the following : [8 × 2 = 16]

a) Define minimal sufficient statistics. Let $X_1, X_2, \ldots, X_n$ be a random sample from $b(1, p)$ distribution. Obtain a minimal sufficient statistics for p.

b) Define power series distribution family. Give an example.

c) State a necessary and sufficient condition for an unbiased estimator to be MVBUE.

d) Define pitman family. Give an example.

e) Define MLE.

f) Define degree of an estimable parameter and kernel.

g) Define loss function and Bayes risk.

h) Define Bayes rule and state Bayes estimator under squared error loss function.

Q2) a) State and prove Basu's theorem.

b) Define complete family. Show that $\{U(0, \theta),\ \theta > 0\}$ is a complete family.

[8 + 8 = 16]

P.T.O.

Q3) a) State and prove Rao-Blackwell theorem.

b) Show that the UMVUE, if exists, is unique.

[8 + 8 = 16]

Q4) a) State and prove Chapman-Robbins-Kiefer inequality.

b) Describe Fisher's scoring method of obtaining an MLE.

[8 + 8 = 16]

Q5) a) Let $X_1, X_2, ..., X_n$ be a random sample from $N(\theta, 1)$, $\theta \in R$, distribution. The prior distribution of θ is $N(0, 1)$. Find the Bayes estimator of θ under absolute error loss function.

b) Describe method of moments estimators. Let $X_1, X_2, ..., X_n$ be a sample from $N(\mu, \sigma^2)$ distribution. Obtain the method of moments estimator of (μ, σ^2).

[8 + 8 = 16]

Q6) a) Show that if T_1 and T_2 are two sufficient statistics, then T_1 is a function of T_2.

b) State and prove Lehmann-Scheffe theorem.

c) Let $X_1, X_2, ..., X_n$ be a random sample from $U\left[\theta-\frac{1}{2},\theta+\frac{1}{2}\right]$ distribution. Obtain an MLE of θ.

d) Let $X \sim P(\lambda)$. $L(\lambda, d(x)) = (\lambda - d(x))^2$. The prior distribution of λ is $G(\alpha, \beta)$. Calculate the Bayes risk for $d(x) = x$.

[4 × 4 = 16]

Q7) Write short notes on the following. [4 × 4 = 16]

a) Curved exponential family

b) The regularity conditions of CR inequality

c) Method of minimum chi-square

d) U-statistic

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Estimation

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Total No. of Pages : 2

Seat No.	

M.Sc. (Part - I) (Semester - I) (CBCS) (Revised) Examination, November - 2019
STATISTICS/APPLIED STATISTICS AND INFORMATION (Paper - IV)
Estimation Theory
Sub. Code : 68231/71448

Day and Date : Friday, 29 - 11 - 2019
Time : 11.00 a.m. to 2.00 p.m.

Total Marks : 80

Instructions :
1) Question No.1 is compulsory.
2) Attempt any four questions from question numbers 2 to 7.
3) Figures to the right indicate full marks.

Q1) Answer the following: [8×2]

a) Define Fisher information function.

b) State invariance property of MLE.

c) Define minimal sufficient statistic.

d) Define minimum variance unbiased estimator.

e) Define ancillary statistics.

f) Define complete sufficient statistic.

g) State Rao-Blackwell theorem.

h) State U-statistics theorem for one sample.

Q2) a) Describe curved exponential family.

b) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$. Obtain Fisher information matrix $I_X(\mu, \sigma^2)$.

c) Discuss Pitman family of distributions in detail. (4+8+4

P.T.O.

Q3) a) State and Prove Crammer-Rao inequality stating the regularity conditions.

b) State and prove Basu's theorem.

[8+8]

Q4) a) Explain method of moment estimators. Obtain moment estimators of a and b of U(a, b) distribution based on random sample of size n.

b) Let X~ Geometric (p) distribution. Obtain UMVUE of p based on a random sample of size n.

[8+8]

Q5) a) What is meant by conjugate prior? Assuming conjugate prior for parameter θ of $N(\theta,1)$. obtain Bayes estimator for θ, under squared error loss function.

b) State and Prove Lehman-Scheffe theorem.

[8+8]

Q6) a) Describe the method of scoring for obtaining maximum likelihood estimate of a parameter function.

b) Let $X_1, X_2, \ldots X_n$ be a random sample from $N(\theta, \theta)$ distribution, $0<\theta<\infty$. Obtain MLE of θ.

[8+8]

Q7) Write short notes on following: [4×4]

a) Bhattacharya Bounds.

b) Bayes estimation under absolute error loss function.

c) Conjugate prior and non informative prior.

d) Sufficient statistic and Neyman factorization theorem.

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Seat No. 01163

M.Sc. (Part - I) (Semester - I) (CBCS) Examination, November - 2019

STATISTICS/APPLIED STATISTICS AND INFORMATICS

Estimation Theory (Paper - IV) (Revised)

Sub. Code : 74910/74977

Day and Date : Friday, 29 - 11 - 2019 **Total Marks : 80**

Time : 11.00 a.m. to 02.00 p.m.

Instructions :
1) Question No. 1 is compulsory.
2) Attempt any four questions from question numbers 2 to 7.
3) Figures to the right indicate full marks.

Q1) Answer the following : [8 × 2 = 16]

a) Define minimal sufficient statistics. Let $X_1, X_2, ..., X_n$ be a random sample from $b(1, p)$ distribution. Obtain a minimal sufficient statistics for p.

b) Define power series distribution family. Give an example.

c) State a necessary and sufficient condition for an unbiased estimator to be MVBUE.

d) Define pitman family. Give an example.

e) Define MLE.

f) Define degree of an estimable parameter and kernel.

g) Define loss function and Bayes risk.

h) Define Bayes rule and state Bayes estimator under squared error loss function.

Q2) a) State and prove Basu's theorem.

b) Define complete family. Show that $\{U(0, \theta), \theta > 0\}$ is a complete family.

[8 + 8 = 16]

Q3) a) State and prove Rao-Blackwell theorem.

b) Show that the UMVUE, if exists, is unique.

[8 + 8 = 16]

Q4) a) State and prove Chapman-Robbins-Kiefer inequality.

b) Describe Fisher's scoring method of obtaining an MLE.

[8 + 8 = 16]

Q5) a) Let $X_1, X_2, ..., X_n$ be a random sample from $N(\theta, 1)$, $\theta \in R$, distribution. The prior distribution of θ is $N(0, 1)$. Find the Bayes estimator of θ under absolute error loss function.

b) Describe method of moments estimators. Let $X_1, X_2, ..., X_n$ be a sample from $N(\mu, \sigma^2)$ distribution. Obtain the method of moments estimator of (μ, σ^2).

[8 + 8 = 16]

Q6) a) Show that if T_1 and T_2 are two sufficient statistics, then T_1 is a function of T_2.

b) State and prove Lehmann-Scheffe theorem.

c) Let $X_1, X_2, ..., X_n$ be a random sample from $U\left[\theta-\frac{1}{2}, \theta+\frac{1}{2}\right]$ distribution. Obtain an MLE of θ.

d) Let $X \sim P(\lambda)$. $L(\lambda, d(x)) = (\lambda - d(x))^2$. The prior distribution of λ is $G(\alpha, \beta)$. Calculate the Bayes risk for $d(x) = x$.

[4 × 4 = 16]

Q7) Write short notes on the following. [4 × 4 = 16]

a) Curved exponential family

b) The regularity conditions of CR inequality

c) Method of minimum chi-square

d) U-statistic