SLR-ES-377

Seat No.	

Set P

M.Sc. (Semester - III) (CBCS) Examination March/April-2019
Statistics
ASYMPTOTIC INFERENCE

Day & Date: Saturday, 27-04-2019 Max. Marks: 70
Time: 03:30 PM to 06:00 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Choose the correct alternative : 14

1) The most common method used to show the asymptotic normality of random variable is ______.
   a) Chebychev's inequality b) Weak law of large numbers
   c) Central limit theorem d) None of these

2) Exponential distribution with location θ with p.d.f

$$f(x,\theta) = \exp\{-(x-\theta)\}, x > \theta, \theta \in \mathrm{R}$$

   a) member of cramer family
   b) member of one parameter exponential family
   c) both (a) and (b)
   d) neither (a) nor (b)

3) For Cauchy distribution with location θ, the consistent estimator of θ is ______.
   a) sample mean b) sample variance
   c) sample median d) none of these

4) If T_n is consistent estimator of then θ then $\left(\frac{5n+1}{n}\right)T_n$ is consistent estimator of ______.
   a) 5 θ b) 6 θ
   c) 0 d) θ

5) Test based on score functions was proposed by ______.
   a) Rao b) Wald
   c) Pearson d) Bartlett

6) Kullback-Leible information index $\mathrm{I}\,(\theta,\theta_0)$ gives ______.
   a) average information per unit observation
   b) information in single observation
   c) information in n observations
   d) None of these

7) An estimator $\overline{\mathrm{X}}_n$ based on a sample of size n from $\mathrm{B}(1,\theta)$ distribution is ______ for θ.
   a) unbiased b) consistent
   c) CAN d) All the above

8) For distribution belonging to one parameter exponential family, MLE of θ is CAN for θ with asymptotic variance equal to ______.
   a) $\mathrm{I}(\theta)$ b) $n\mathrm{I}(\theta)$
   c) $\frac{1}{n\mathrm{I}(\theta)}$ d) $\frac{1}{\mathrm{I}(\theta)}$

9) Variance stabilizing transformation for normal population is ______.
   a) logarithmic
   b) square root
   c) $\tan h^{-1}$
   d) $\sin^{-1}$

10) If T_n is consistent estimator of θ then _______.
   a) T_n is also a consistent estimator of θ^2
   b) T_n^2 is also a consistent estimator of θ
   c) T_n^2 Is also a consistent estimator of θ^2
   d) None of the above

11) In a random sample of size n from Poisson distribution with mean λ, MLE of λ is reported to be 2. Then variance of asymptotic normal distribution of $\sqrt{n}\,(e^{-\bar{X}_n} - e^{-\lambda})$ is estimated by ______.
   a) $4\,e^{-4}$
   b) $2\,e^{-4}$
   c) $2\,e^{-2}$
   d) e^{-4}

12) In case of $N(\omega, \sigma^2), \mu \in R, \sigma > 0$, the MLE of (ω, σ^2) is _______.
   a) unbiased and consistent
   b) asymptotically unbiased and consistent
   c) unbiased and not consistent
   d) asymptotically unbiased and not consistent

13) Let $X_1, X_2, \ldots\ldots, X_n$ be iid exponential with mean θ then ________.
   a) $\bar{X}_n$ is consistent estimator of θ
   b) $\frac{1}{\bar{X}_n}$ is consistent estimator of $\frac{1}{\theta}$
   c) $e^{-t/\bar{X}_n}$ is consistent estimator of $e^{-t/\theta}$
   d) All the above

14) Based on random sample of size n from $U\,(0, \theta), \theta > 0$, CAN estimator of θ is _____.
   a) $\bar{X}_n$
   b) $2\,\bar{X}_n$
   c) $X_{(n)}$
   d) $2\,X_{(n)}$

Q.2 A) Attempt any four of the following : 08

1) Define consistent estimator and give an example of estimator which is not consistent.
2) What is variance stabilizing transformation?
3) Define super efficient estimator.
4) Define likelihood ratio test (LRT).
5) Define one parameter exponential family of distributions.

B) Write short notes on any two of the following. 06

1) Comparison of consistent estimators
2) Asymptotic relative efficiency
3) Bartlett's test for homogeneity of variances

Q.3 A) Attempt any two of the following : 08

1) Show that joint consistency is equivalent to marginal consistency.
2) Give two examples for multiparameter exponential family.
3) Let $X_1, X_2, \ldots\ldots, X_n$ be iid $N(\theta, \theta)$ for some $\theta > 0$. Find consistent estimator of θ^2

B) Answer any one of the following question. 06

1) Describe variance stabilizing transformation for exponential distribution with mean θ.
2) Let $X_1, X_2, \ldots\ldots, X_n$, be iid poisson (θ). Show that $\bar{X}_n$ is CAN for θ. Let $\Psi(\theta) = \theta e^{-\theta}$. Show that $\bar{X}_n\, e^{-\bar{X}_n}$ is CAN for $\psi(\theta)$for all values of θ except at $\theta = 1$. What is the asymptotic distribution of $\bar{X}_n\, e^{-\bar{X}_n}$ at $\theta = 1$?

Q.4 A) Answer any two of the following questions. **10**

1) State and prove invariance property of CAN estimator.
2) Let $X_1, X_2, \ldots\ldots, X_n$ be iid exponential with location θ .Show that X_1 is consistent but not CAN for θ.
3) Let $X_1, X_2, \ldots\ldots, X_n$ be iid from $N(\theta, \theta^2)$. Obtain $100\ (1-\alpha)\%$ confidence interval for θ using variance stabilizing transformation.

B) Answer any one of the following question. **04**

1) Prove or disprove: consistent estimator is always CAN.
2) Let $X_1, X_2, \ldots\ldots, X_n$ be iid exponential with mean θ. Obtain consistent estimator for the median and hence obtain the consistent estimator for mean of the distribution.

Q.5 Answer any two of the following questions. **14**

a) State Cramer-regularity conditions and Cramer-Huzurbazar results.

b) Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample of size n from the distribution having p.d.f.

$$f(x, \omega, \lambda) = \frac{1}{\lambda}\, e^{-\left(\frac{x-\omega}{\lambda}\right)}\ x \geq \omega, \lambda > 0$$

Obtain moment estimates of (ω, λ) and its asymptotic variance-covariance matrix.

c) Derive the asymptotic distribution of Chi-square test for goodness of fit.

SLR-MM – 521

Seat No.	

M.Sc. – II (Semester – III) (CGPA) Examination, 2015
STATISTICS (Paper No. – XI)
Asymptotic Inference (New)

Day and Date : Monday, 16-11-2015 Total Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and **2** are **compulsory**.*
*3) Attempt **any three** from Q. **3** to **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternatives of the following questions.

i) Let T_n be the sequence of consistent estimators of θ, then

a) $\lim_{n\to\infty} Var(T_n) = 0$ b) $\lim_{n\to\infty} E(T_n - \theta)^2 = 0$

c) both a and b d) neither a nor b

ii) Let $X_1, X_2 ... X_n$ be iid from B (1, θ), then asymptotic distribution of $\overline{X}(1-\overline{X})$ is

a) Normal with mean $\theta(1-\theta)$ and variance $\frac{\theta(1-\theta)}{n}(1-2\theta)^2$, if $\theta \neq \frac{1}{2}$

b) Normal with mean $\theta(1-\theta)$ and variance $\frac{\theta(1-\theta)}{n}(1-2\theta)^2$, for all θ

c) Normal with mean $\theta(1-\theta)$ and variance $\frac{\theta(1-\theta)}{n}$, for all θ

d) None of the above

iii) Let $X_1, X_2 ... X_n$ be iid from $N(\theta, \sigma^2)$, then

a) $\overline{X}$ is unique consistent estimator of θ

b) $\overline{X}$ and sample median are the only two consistent estimators for θ

c) No consistent estimator exist for θ if σ^2 is unknown

d) There are infinitely many consistent estimators for θ

iv) Let $X_1, X_2...X_n$ be iid from $U(0, \theta)$ then

a) $X_{(n)}$ is CAN

b) $X_{(n)}$ is consistent and but not asymptotically normal

c) $X_{(n)}$ is consistent and BAN

d) None of the above

v) Let $X_1, X_2...X_n$ be iid $\exp(\beta, \sigma)$, then

a) $X_{(1)}$ is MLE of β

b) S^2 is MLE of σ

c) $X_{(1)}$ is CAN estimator β

d) MLE of σ does not exists

B) Fill in the blanks.

i) Let $X_1, X_2...X_n$ be iid from Cauchy (θ, 1) distribution, then consistent estimator of θ is ______________.

ii) Asymptotic distribution of sample distribution function is ________ provided underlying distribution is continuous.

iii) If distribution of X belong to one parameter exponential family of distributions then the moment estimator of θ based on sufficient statistic is __________ estimator.

iv) Wald test statistic for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$ is _________.

v) Asymptotic variance of super efficient estimator is _______ than fisher lower bound for some $\theta = \theta_0$ and equal to fisher lower bound otherwise.

C) State whether following statements are **true** or **false**.

i) Let $X_1, X_2...X_n$ be iid from $P(\theta)$, then $\overline{X}$ is unique consistent estimator of θ.

ii) Let $X_1, X_2...X_n$ be iid from $f(x, \theta)$, and $T = T(X_1, X_2...X_n)$ be a CAN estimator for θ, if g (.) is continuous, differentiable function the g (T) is CAN estimator for $g(\theta)$ provided $\frac{dg(\theta)}{d\theta} \neq 0$.

iii) Cramer family of distributions is subset of exponential family of distribution.

iv) Log transformation is the variance stabilizing transformation for a $\exp(\theta)$ population. **(5+5+4)**

2. a) Write a note on method of moments estimation for CAN estimator.

   b) Let $X_1, X_2 ... X_n$ be iid $\exp(\theta, \sigma)$, then show that $\left(X_{(1)}, \sum_{i=1}^{n}(X_i - X_{(1)})\right)'$ is jointly consistent for $(\theta, \sigma)'$.

   c) Describe Rao's score test.

   d) Write a note on super efficient estimator. **(4+4+3+3)**

3. a) Define weak and strong consistency. Obtain consistent estimator for mean of double exponential distribution based on sample of size n.

   b) Define asymptotic relative efficiency. Let $X_1, X_2 ... X_n$ be iid from $U(0, \theta)$, obtain relative efficiency of $X_{(n)}$ to $2\overline{X}$. **(7+7)**

4. a) Define one parameter Cramer family of distributions and show that in one parameter Cramer family, with probability approaches to 1 as $n \to \infty$, the likelihood equation admits consistent solution.

   b) Let $X_1, X_2 ... X_n$ be iid $N(\theta, \theta)$, then obtain three consistent estimators for θ. **(10+4)**

5. a) Let $X_1, X_2 ... X_n$ be iid Cauchy $(\theta, 1)$, then find the asymptotic distribution MLE of θ and asymptotic variance.

   b) Let $X_1, X_2 ... X_n$ be iid $\exp(\theta)$, θ mean, using show that $\overline{X}$ is CAN estimator for θ and obtain a CAN estimator for $P(X > t)$ and its asymptotic variance. **(7+7)**

6. a) Let $X_1, X_2 ... X_n$ be iid $B(1, \theta)$. Construct $100(1 - \alpha)$ level VST confidence interval for θ.

   b) Let $X_1, X_2 ... X_n$ be iid $N(\theta, 1)$, then derive LRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$. **(7+7)**

7. a) Derive Bartlett's test for testing equality variances of a normal populations.

   b) Let X be a multinomial vector with 4 cells and cell probabilities are $P(c_1) = \theta^2, P(c_2) = P(c_3) = \theta(1-\theta), P(c_4) = (1-\theta)^2$. Obtain the MLE and discuss CAN property of the same. **(7+7)**

Seat No.	

M.Sc. – II (Semester – III) Examination, 2016
STATISTICS (Paper – XI)
Asymptotic Inference (New) (CGPA)

Day and Date : Tuesday, 29-3-2016 Max. Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternatives of the following questions :

i) Let $X_1, X_2 \ldots X_n$ be iid exponential r. v. with mean θ. Then the asymptotic distribution of $\sqrt{n}\left(\frac{\bar{x}}{s}-1\right)$, where $s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$ is given by

a) N (0, 1) b) N (0, θ) c) N (1, 1) d) Exp (θ)

ii) Let $X_1, X_2 \ldots X_n$ be iid $U\left(\theta-\frac{1}{2},\theta+\frac{1}{2}\right)$ r. v.'s, then maximum likelihood estimator for θ is

a) $\bar{x}$ b) $\frac{x_{(1)}+x_{(n)}}{2}$ c) $\frac{x_{(1)}+x_{(n)}}{4}$ d) $x_{(1)} + 1$

iii) Let $X_1, X_2 \ldots X_n$ be iid from N (θ, 1), then which of the following is not correct ?

a) $\bar{X}$ is consistent estimator for θ

b) $\bar{X}$ is BAN estimator for θ

c) Sample median is consistent θ

d) Sample median is BAN θ

iv) Which of the following is not true ?

a) Sample mean is always consistent estimator for population mean, if exists

b) Sample percentiles are always consistent estimator for population percentiles

c) Cauchy distribution belong to one parameter exponential family

d) None of the above

v) Which of the following distribution not belongs to Cramer family of distribution ?

a) DE (a, b), both a and b are unknown

b) DE (a, b), a is known but b is unknown

c) Cauchy (a, b), both a and b are unknown

d) Exp (1, b), b is unknown

B) Fill in the blanks :

i) Let $X_1, X_2 \ldots X_n$ be iid from Poisson distribution with mean θ, a consistent estimator for P(X=1) is __________.

ii) Let $X_1, X_2 \ldots X_n$ be iid from Exp $\left(\text{mean}\,\frac{1}{\theta}\right)$. Then BAN estimator for θ is __________.

iii) Let $X_1, X_2 \ldots X_n$ be iid from f (x, θ), then g(x) is consistent estimator for $g(\theta)$, provided __________.

iv) __________ is the example of consistent estimator which is not asymptotic normal.

v) The asymptotic distribution of LRT is __________.

C) State whether following statements are **true** or **false** :

i) Every unbiased estimator is consistent estimator

ii) Variance of super efficient estimator is less than equal to fisher lower bound

iii) Let $X_1, X_2 \ldots X_n$ be iid from N $(0, \theta)$, θ unknown, then consistent estimator for θ not exist

iv) Square root transformation is the variance stabilizing transformation for a binomial population. **(5+5+4)**

2. a) State and prove invariance property of consistent estimator.
   b) Write a note on marginal and joint consistency.
   c) Describe Rao's score test.
   d) Show that sample mean is always CAN estimator for population mean if exists. **(4+4+3+3)**

3. a) Examine whether S_n^2 and S_{n-1}^2 are consistent estimators of normal variance σ^2, assuming that the normal mean is unknown.
   b) Let $X_1, X_2, \ldots X_n$ be iid from distribution with pdf $f(x, \theta) = \frac{\theta}{x^{(\theta-1)}}, x \geq 1, \theta \geq 0$, then obtain consistent estimator for θ. **(7+7)**

4. a) Let $X_1, X_2, \ldots X_n$ be iid from distribution with pdf $f(x, \theta), \theta \in \theta$, obtain asymptotic distribution of sample percentile.
   b) Show that, in exponential family of distribution asymptotic distribution of moment estimator based on sufficient statistic is normal with asymptotic variance $\frac{1}{I_n(\theta)}$. **(7+7)**

5. a) What is variance stabilising transformation and explain its use of constructing large sample confidence intervals.
   b) Obtain 100 $(1 - \alpha)$ level asymptotic confidence interval for the mean of Poisson distribution. **(7+7)**

6. a) State Cramer's theorem and prove that likelihood equation admits consistent solution.
   b) Let $X_1, X_2, \ldots X_n$ be iid from $B(1, \theta)$, show that $\overline{X}$ is CAN for θ and check whether it is BAN. **(7+7)**

7. a) Derive the asymptotic null distribution of the likelihood ratio test statistic.
   b) Explain Person test for goodness of fit. **(7+7)**

SLR-JS-378

Seat No.	

Set P

M.Sc. (Semester - III) (CBCS) Examination Oct/Nov-2019
Statistics
ASYMPTOTIC INFERENCE

Day & Date: Monday, 18-11-2019 Max. Marks: 70
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. **14**

1) The criterion used to choose between two consistent estimators is ____.
   a) Smallness of mean
   b) Smallness of variance
   c) Smallness of mean squared error
   d) None of these

2) $\{\cup\ (o, \theta), \theta > 0\} =$ is _____.
   a) one parameter exponential family
   b) cramer family
   c) both (a) and (b)
   d) neither (a) nor (b)

3) If T_n is consistent for θ then g (T_n) is consistent for g (θ) if _____.
   a) g is linear function
   b) g is continuous function
   c) g is differentiable function
   d) none of these

4) Given a random sample of size n from $N(\theta, 1)$, the estimator $\overline{X}_n$ is _____ for θ.
   a) unbiased
   b) consistent
   c) CAN
   d) all the above

5) The test used to investigate the homogeneity of variances of several normally distributed populations is _____.
   a) Rao test
   b) Bartlett test
   c) Pearson test
   d) Wald test

6) Kullback - Leibie information index _____.
   a) $I(\theta, \theta_0) < 0$
   b) $I(\theta, \theta_0) > 0$
   c) $I(\theta, \theta_0) \geq 0$
   d) $I(\theta, \theta_0) = 0$

7) In case of $U(0, \theta), \theta > 0$ the MLE of θ is_____.
   a) unbiased and consistent
   b) asymptotically unbiased and consistent
   c) unbiased but not consistent
   d) asymptotically unbiased but not consistent

8) For distribution belonging to one parameter exponential family, moment estimator of θ based on sufficient statistic is CAN for θ with asymptotic variance _____.
   a) $n\, I(\theta)$
   b) $\frac{1}{nI(\theta)}$
   c) $I(\theta)$
   d) $\frac{1}{I(\theta)}$

9) Variance stabilizing transformation for poisson population is _____.
   a) square root
   b) logarithmic
   c) sin^{-1}
   d) $\tan h^{-1}$

10) If T_n is consistent estimator of θ then e^{T_n} is _____.
   a) unbiased estimator of e^{θ}
   b) consistent estimator of e^{θ}
   c) MVU estimator of e^{θ}
   d) none of the above

11) Let $x_1, x_2, \ldots, x_n$ be iid with $E(xi^2) = V(xi) = \sigma^2$ then asymptotic distribution of $\overline{X}_n$ is _____.
   a) $N(0,1)$
   b) $N(0,\sigma^2)$
   c) $N\left(0,\frac{1}{n}\right)$
   d) $N\left(0\frac{\sigma^2}{n}\right)$

12) The sample median is consistent estimator for θ in the case of ____.
   a) $N(\theta,1)$
   b) $U(\theta-1,\ \theta+1)$
   c) $Laplace(\theta,1)$
   d) all the above

13) Let $x_1, x_2, \ldots, x_n$ be iid $N(\mu,1)$. Then asymptotic distribution of sample median M_n is _____.
   a) $N\left(\mu,\frac{\pi}{n}\right)$
   b) $N\left(\mu,\frac{\pi}{2n}\right)$
   c) $N\left(\mu,\frac{\pi^2}{4n}\right)$
   d) $N\left(\mu,\frac{1}{n}\right)$

14) With sufficiently large sample size with probability close to one, the likelihood equation admits _____.
   a) unique consistent solution
   b) two consistent solution
   c) more than two consistent solutions
   d) none of these

Q.2 A) Answer the following questions. (Any Four) **08**
1) Define strong consistency.
2) Define Rao's score test.
3) Define BAN estimator.
4) Define mulitparameter exponential family.
5) Define asymptotic relative efficiency.

B) Write Notes. (Any Two) **06**
1) Super efficient estimator
2) CAN estimation in multiparameter exponential family
3) Bartlett's test for homogeneity of variances

Q.3 A) Answer the following questions. (Any Two) **08**
1) Show that sample variance is consistent estimator of population variance, if it exists.
2) Show that sample distribution function at a given point is CAN for the population distribution function at the same point.
3) Let $x_1, x_2 \ldots\ldots, x_n$ be iid from exponential distribution with location parameter θ. Examine whether $x_{(1)}$ is consistent estimator for θ.

B) Answer the following questions. (Any One) **06**

1) Describe variance stabilizing transformation for pisson population.
2) Let x_1, x_2 ……., x_n be iid $B(1, \theta)$. Show that $\overline{X}_n$ is CAN for θ. Let $\psi(\theta) = \theta(1-\theta)$. Show that $\overline{X}_n(1-\overline{X}_n)$ is CAN for $\psi(\theta)$ for all values of $\theta\ except\ \theta = \frac{1}{2}$. What is asymptotic distribution of $\overline{X}_n(1-\overline{X}_n)$ at $\theta = \frac{1}{2}$?

Q.4 A) Answer the following questions. (Any Two) **10**

1) In case of one parameter exponential family, show that moment estimator based on sufficient statistic is CAN for the parameter.
2) Let x_1, x_2 ……., x_n be iid with distribution having p.d.f. $f(x, \theta) = \frac{\theta}{x^{\theta+1}}$, $x > 1, \theta > 0$. Obtain CAN estimator of θ.
3) Let x_1, x_2 ……., x_n be iid from $N(\theta, \theta)$, for $\theta > 0$. Obtain $100(1-\propto)\%$ confidence interval for θ using variance stabilizing transformation.

B) Answer the following questions. (Any One) **04**

1) Explain with illustration that the MLE need not be CAN.
2) Let x_1, x_2 ……., x_n be iid exponential with mean θ. Obtain consistent estimator for first and third quartile of the distribution.

Q.5 Answer the following questions. (Any Two) **14**

a) Under Cramer - Huzurbazar regularity conditions, show that the likelihood equation admits a solution which is consistent.

b) Let x_1, x_2 ……., x_n be a random sample of size n from $N(\mu, \sigma^2)$. Obtain MLE of (μ, σ^2). Show that it is CAN for (μ, σ^2). Obtain its asymptotic variance covariance matrix.

c) Derive the asymptotic distribution of likelihood ratio statistic.

SLR-BP – 487

Seat No.	

M.Sc. (Part – II) (Semester – III) Examination, 2015
STATISTICS (Paper – XI)
Asymptotic Inference

Day and Date : Wednesday, 15-4-2015 Total Marks : 70

Time : 3.00 p.m. to 6.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q.No. (**1**) and Q.No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**)*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) An estimator $\hat{\mu}_Y$ of population value μ_Y is more efficient when compared with another estimator $\tilde{\mu}_Y$ if

a) $E(\hat{\mu}_Y) > E(\tilde{\mu}_Y)$

b) It has smaller variance

c) Its cdf is flatter than that of the other

d) Both estimators are unbiased and $Var(\hat{\mu}_Y) < Var(\tilde{\mu}_Y)$

2) The variance stabilizing transformation for binomial population is

a) square root b) logarithmic c) $\sin^{-1}$ d) $\tanh^{-1}$

3) In case of $N(\mu, \sigma^2), \mu \in R, \sigma^2 > 0$, the MLE of σ^2 is

a) unbiased and consistent b) biased and consistent

c) unbiased and not consistent d) biased and not consistent

4) IF T_n is consistent estimator of θ then $\varphi(T_n)$ is consistent estimator of $\varphi(\theta)$ if

a) φ is linear function b) φ is continuous function

c) φ is differentiable function d) none of these

5) The asymptotic distribution of Rao's statistic is

a) normal b) t c) chi-square d) F

B) Fill in the blanks. **5**

1) Let $X_1, X_2, ..., X_n$ be iid with $E\left(X_i^2\right) = Var(X_i) = \sigma^2$. The asymptotic distribution of $\overline{X}_n$ is ________ .

2) Let $X_1, X_2, ..., X_n$ be iid $B(1, \theta)$. CAN estimator of P_θ $(X = 1)$ is ________.

3) Cramer class is _________ than exponential class of distributions.

4) For Laplace $(\theta, 1)$ distribution, asymptotic variance of $\overline{X}_n$ is _________.

5) Let $X_1, X_2,...,X_n$ be iid $N(\theta, 1)$. Then CAN estimator of θ^2 is ________.

C) State whether the following statements are **true** or **false**. **4**

1) Cauchy distribution is a member of Cramer family.

2) Every CAN estimator is consistent.

3) Consistency of estimator is always unique.

4) Consistent estimator based on MLE need not be CAN.

2. a) Answer the following. **6**

i) State Cramer-Huzurbazar results.

ii) Let $X_1, X_2, ..., X_n$ be iid $U(0, \theta)$. By computing the actual probability, show that $X_{(n)}$ is consistent estimator for parameter θ.

b) Write short notes on the following. **8**

i) Super efficient estimator.

ii) Strong consistency.

3. a) Define consistent estimator. State and prove invariance property of consistent estimator.

b) Let $X_1, X_2, ..., X_n$ be a random sample from exponential distribution with location parameter θ. Find two consistent estimators of θ. Examine the CAN property of the suggested estimators. **(6+8)**

4. a) State Cramer regularity conditions in one parameter set up. Give an example of a distribution which satisfies Cramer regularity conditions. Justify your answer.

   b) Let $X_1, X_2, ..., X_n$ be a sample from $N(\theta, \sigma^2)$ distribution. Obtain CAN estimator of σ^2. **(7+7)**

5. a) Define CAN estimator. Show that sample distribution function at a given point is CAN for the population distribution function at the same point.

   b) Let $X_1, X_2, ..., X_n$ be iid $N(\theta, \sigma^2)$ random variables. Find the variance stabilizing transformation for S^2 and obtain $100(1-\alpha)\%$ confidence interval for σ^2 based on the transformation. **(7+7)**

6. a) Describe Bartlett test for homogeneity of variances.

   b) Let $X_1, X_2, ... X_n$ be a random sample of size n form the distribution with pdf,

   $f(x; \mu, \lambda) = \frac{1}{\lambda} \exp\left[-\left(\frac{x-\mu}{\lambda}\right)\right], x \geq \mu, \lambda > 0$. Obtain moment estimator of (μ, λ) and its variance-covariance matrix. **(6+8)**

7. a) Define m-parameter exponential family of distributions. Show that $\{N(\mu, \sigma^2), \mu \in R, \sigma^2 > 0\}$ is a two-parameter exponential family.

   b) Let $X_1, X_2, ... X_n$ be iid Poisson (λ). Obtain CAN estimator of $\lambda e^{-\lambda}$. Discuss its asymptotic distribution at $\lambda = 1$. **(6+8)**

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SLR-BP – 487

Seat No.	

M.Sc. (Part – II) (Semester – III) Examination, 2015
STATISTICS (Paper – XI)
Asymptotic Inference

Day and Date : Wednesday, 15-4-2015 Total Marks : 70

Time : 3.00 p.m. to 6.00 p.m.

Instructions : 1) *Attempt **five** questions.*
2) *Q.No. (**1**) and Q.No. (**2**) are **compulsory**.*
3) *Attempt **any three** from Q. No. (**3**) to Q. No. (**7**)*
4) *Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) An estimator $\hat{\mu}_Y$ of population value μ_Y is more efficient when compared with another estimator $\tilde{\mu}_Y$ if

a) $E(\hat{\mu}_Y) > E(\tilde{\mu}_Y)$

b) It has smaller variance

c) Its cdf is flatter than that of the other

d) Both estimators are unbiased and $Var(\hat{\mu}_Y) < Var(\tilde{\mu}_Y)$

2) The variance stabilizing transformation for binomial population is

a) square root b) logarithmic c) $\sin^{-1}$ d) $\tanh^{-1}$

3) In case of $N(\mu, \sigma^2), \mu \in R, \sigma^2 > 0$, the MLE of σ^2 is

a) unbiased and consistent b) biased and consistent

c) unbiased and not consistent d) biased and not consistent

4) IF T_n is consistent estimator of θ then $\varphi(T_n)$ is consistent estimator of $\varphi(\theta)$ if

a) φ is linear function b) φ is continuous function

c) φ is differentiable function d) none of these

5) The asymptotic distribution of Rao's statistic is

a) normal b) t c) chi-square d) F

B) Fill in the blanks. **5**

1) Let $X_1, X_2, ..., X_n$ be iid with $E\left(X_i^2\right) = Var(X_i) = \sigma^2$. The asymptotic distribution of $\overline{X}_n$ is ________ .

2) Let $X_1, X_2, ..., X_n$ be iid $B(1, \theta)$. CAN estimator of P_θ $(X = 1)$ is ________.

3) Cramer class is _________ than exponential class of distributions.

4) For Laplace $(\theta, 1)$ distribution, asymptotic variance of $\overline{X}_n$ is _________.

5) Let $X_1, X_2,...,X_n$ be iid $N(\theta, 1)$. Then CAN estimator of θ^2 is ________.

C) State whether the following statements are **true** or **false**. **4**

1) Cauchy distribution is a member of Cramer family.

2) Every CAN estimator is consistent.

3) Consistency of estimator is always unique.

4) Consistent estimator based on MLE need not be CAN.

2. a) Answer the following. **6**

i) State Cramer-Huzurbazar results.

ii) Let $X_1, X_2, ..., X_n$ be iid $U(0, \theta)$. By computing the actual probability, show that $X_{(n)}$ is consistent estimator for parameter θ.

b) Write short notes on the following. **8**

i) Super efficient estimator.

ii) Strong consistency.

3. a) Define consistent estimator. State and prove invariance property of consistent estimator.

b) Let $X_1, X_2, ..., X_n$ be a random sample from exponential distribution with location parameter θ. Find two consistent estimators of θ. Examine the CAN property of the suggested estimators. **(6+8)**

4. a) State Cramer regularity conditions in one parameter set up. Give an example of a distribution which satisfies Cramer regularity conditions. Justify your answer.

   b) Let $X_1, X_2, \ldots, X_n$ be a sample from $N(\theta, \sigma^2)$ distribution. Obtain CAN estimator of σ^2. **(7+7)**

5. a) Define CAN estimator. Show that sample distribution function at a given point is CAN for the population distribution function at the same point.

   b) Let $X_1, X_2, \ldots, X_n$ be iid $N(\theta, \sigma^2)$ random variables. Find the variance stabilizing transformation for S^2 and obtain $100(1-\alpha)\%$ confidence interval for σ^2 based on the transformation. **(7+7)**

6. a) Describe Bartlett test for homogeneity of variances.

   b) Let $X_1, X_2, \ldots X_n$ be a random sample of size n form the distribution with pdf,

   $f(x;\mu,\lambda) = \frac{1}{\lambda}\exp\left[-\left(\frac{x-\mu}{\lambda}\right)\right], x \geq \mu, \lambda > 0$. Obtain moment estimator of (μ, λ) and its variance-covariance matrix. **(6+8)**

7. a) Define m-parameter exponential family of distributions. Show that $\{N(\mu,\sigma^2), \mu \in R, \sigma^2 > 0\}$ is a two-parameter exponential family.

   b) Let $X_1, X_2, \ldots X_n$ be iid Poisson (λ). Obtain CAN estimator of $\lambda e^{-\lambda}$. Discuss its asymptotic distribution at $\lambda = 1$. **(6+8)**

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