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M.Sc. (Part – II) (Semester – III) Examination, 2015
STATISTICS (Paper – XIII)
Planning and Analysis of Industrial Experiments (New CGPA)

Day and Date : Friday, 20-11-2015
Time : 2.30 p.m. to 5.00 p.m.

Total Marks : 70

- Instructions :** 1) Attempt **five** questions.
2) Q. No. (1) and Q. No. (2) are **compulsory**.
3) Attempt **any three** from Q. No. (3) to Q. No. (7).
4) Figures to the **right** indicate **full** marks.

1. A) Choose the correct alternative : 5
- 1) Smaller the experimental error _____ efficient the design.
a) less b) more
c) not d) none of the above
 - 2) If AB and BC are confounded with incomplete block in 2^n experiment, then automatically confounded effect is
a) ABC b) AC c) A d) B
 - 3) The degrees of freedom corresponding to error in single replicate design is
a) 0 b) 1
c) 2 d) None of above
 - 4) Confounding is necessary to reduce
a) Block size b) No. of blocks
c) No. of factors d) All of above
 - 5) Fractional factorial experiment reduces
a) factors b) levels of factors
c) both a) and b) d) neither a) nor b)



- B) Fill in the blanks : 5
- 1) In factorial experiment one can estimate _____ and _____ effects.
 - 2) The shortest word length in defining relation is called as _____
 - 3) Variables which are hard to control are called _____
 - 4) In 3^3 experiment with factors A, B and C the interaction AB has _____ d.f.
 - 5) Preferably _____ interaction is chosen for confounding.
- C) State whether the following statements are **true** or **false** : 4
- 1) In 2^3 design, generally we choose ABC as confounding factor.
 - 2) Experimental error is due to experimenter's mistake.
 - 3) For 2^k design the complete model would contain 2^{k-2} effects.
 - 4) In Response Surface Study the factors must be quantitative.
2. a) Define with one example : 6
- i) Minimum aberration design.
 - ii) Resolution of factorial design.
- b) Write short notes on the following : 8
- i) Yates table for 2^3 factorial experiments.
 - ii) Central Composite Design.
3. a) Describe the random effect model of one-way classification.
- b) Describe Taguchi arrays. (7+7)
4. a) Explain $\frac{1}{4}$ fraction of 2^k design with suitable example.
- b) Write down lay-out of 2^4 confounded design with higher order interaction is confounded. (8+6)
5. a) Explain advantages and disadvantages of confounding.
- b) Explain partial confounding with illustration. (7+7)



6. a) Explain Response Surface methodology.
- b) Define :
- i) Principle fraction
 - ii) Aliases sets
 - iii) Clearly estimate effects. **(7+7)**
7. a) Explain analysis of 2^n factorial experiment in 'r' replicates.
- b) Describe basic principles of Design of Experiments. **(7+7)**
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Seat
No.

M.Sc. (Part – I) (Sem. – II) Examination, 2015
(Old – CGPA)
STATISTICS (Paper – VII)
Linear Models and Design of Experiments

Day and Date : Thursday, 19-11-2015

Total Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions: 1) Attempt **five** questions.

2) Q. No. 1 and Q. No. 2 are **compulsory**.

3) Attempt **any three** from Q. No. 3 to Q. No. 7.

4) Figures to the **right** indicate **full** marks.

1. A) Select the correct alternative :

5

1) Let $E(Y_1) = \theta_1 + \theta_2$, $E(Y_2) = \theta_1 - \theta_2$, $E(Y_3) = \theta_1$, $V(Y_i) = \sigma^2$; $i = 1, 2, 3$. Let

$\hat{\theta}_i$ be the least square estimator of θ_i . Which of the following statement is correct ?

a) $\hat{\theta}_1 = \frac{Y_1 + Y_2 + Y_3}{2}$

b) $\hat{\theta}_2 = \frac{Y_1 + Y_2 + Y_3}{2}$

c) $V(\hat{\theta}_1) = \frac{3\sigma^2}{4}$

d) $V(\hat{\theta}_2) = \frac{\sigma^2}{2}$

2) An experimenter wishes to compare 8 treatments in blocks of size 4, using BIBD with 14 blocks, then any pair of treatments appear together in _____ blocks.

a) 4

b) 3

c) 2

d) 1

3) In a connected block design with v treatments and b blocks, rank of D matrix is

a) $v - 1$ b) $b - 1$ c) $v + 1$ d) $b + 1$



- 4) In the linear model $y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$, $i = 1, 2, 3$, $j = 1, 2, 3, 4$, the number of linearly independent estimable parametric functions is
 a) 12 b) 3 c) 4 d) 7
- 5) From an RBD with 4 treatments and 5 blocks one block is removed, then the resulting design is
 a) CRD b) BIBD c) LSD d) RBD

B) Fill in the blanks :

5

- 1) In a general linear model, the covariance between any linear function belonging to the error space and any BLUE is _____
- 2) The BLUE of a treatment contrast $\sum_i c_i \alpha_i$ in one-way ANOVA model is _____
- 3) The degrees of freedom of error SS in two-way without interaction ANOCOVA model with p rows, q columns, 1 observation per cell and 1 covariate is _____
- 4) In a general block design, covariance between adjusted treatment totals and block totals is _____
- 5) In a general linear model, $\underline{y} = X\underline{\beta} + \underline{\varepsilon}$, a linear parametric function $\underline{\lambda}' \underline{\beta}$ is estimable if and only if $\underline{\lambda}' =$ _____

C) State **true** or **false** :

4

- 1) In a general linear model, the normal equations are always consistent.
 2) Individual parameters are not estimable in one-way and two-way ANOVA models.
 3) A connected design is necessarily balanced.
 4) BIBD is not orthogonal.
2. a) Define BIBD and show that it is connected, non-orthogonal and balanced.
 b) Show that for BIBD with parameters (v, b, r, k, λ) , $b \geq v$. (7+7)
3. Consider $E(Y_1) = \theta_1 + \theta_2 + \theta_3$, $E(Y_2) = E(Y_4) = \theta_1 - \theta_4$, $E(Y_3) = \theta_1 + \theta_2$ and $\text{cov}(\underline{Y}) = \sigma^2 I_n$.
 a) Check whether the above model is full-rank model.
 b) Obtain rank of the estimation space and rank of the error space.
 c) Obtain one solution of normal equations and hence obtain BLUE of $\theta_1 + \theta_2$ if it is an estimable parametric function. (4+4+6)



- 4. a) Explain Tukey’s method of comparison of k different individual means.
 - b) Prove or disprove BLUE is unique.
 - c) For the linear model, $\underline{y} = \underline{X}\underline{\beta} + \underline{\varepsilon}$, explain conditional and unconditional sum of squares of error. **(4+4+6)**
5. a) Given below is the incidence matrix N_A of design A. Check whether the design is connected.

$$N_A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

- b) In a general block design, show that row sums and column sums of C matrix are all zero.
 - c) Obtain BLUE of $\sum_i l_i \alpha_i$, $\sum_i l_i = 0$ in the one-way ANOCOVA model with single covariate. **(3+3+8)**
6. A) Write the linear model for one-way classification with one concomitant variable. Obtain the least square estimates of its parameters. **14**
- B) Derive the test for testing the hypothesis of the equality of row effects in two-way ANOVA without interaction model with one observation per cell.
7. a) Explain the following terms :
- i) Estimation space
 - ii) BIBD and its properties.
- b) In a general linear model $\underline{y} = \underline{X}\underline{\beta} + \underline{\varepsilon}$, develop a test for testing $H_0 : \wedge \underline{\beta} = \underline{0}$. **(8+6)**
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Set P

**M.Sc. (Semester - III) (CBCS) Examination March/April-2019
Statistics**

PLANNING AND ANALYSIS OF INDUSTRIAL EXPERIMENTS

Day & Date: Tuesday, 30-04-2019
Time: 03:30 PM To 06:00 PM

Max. Marks: 70

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Choose Correct Alternative from the following. **14**

- 1) Confounding is necessary to reduce _____.
 a) Block size
 b) No. of blocks
 c) No. of factors
 d) All of these
- 2) For a 2^{6-2} experiment with defining relation I=ABCD=CDEF=ABEF is a _____ design.
 a) Resolution IV
 b) Resolution II
 c) Resolution III
 d) Resolution V
- 3) RBD is _____ orthogonal.
 a) Always
 b) Not
 c) Sometimes
 d) All of these
- 4) If there are five factors each at two levels and are conducted in two replications, then error degrees of freedom are _____.
 a) 0
 b) 31
 c) 32
 d) 63
- 5) In the field experimentation, when experimental material is heterogeneous, we use _____.
 a) CRD
 b) RBD
 c) LSD
 d) All of these
- 6) In one half fraction with I = +ABC is called _____ fraction.
 a) Principal
 b) Alternate
 c) Complementary
 d) Both b and c
- 7) The local control helps to _____.
 a) Reduce the number of treatments
 b) Maintain greater homogeneity of experiment
 c) Increase the number of plots
 d) None of these
- 8) If AB and BC are confounded with incomplete block in 2^n experiment, then automatically confounded effect is _____.
 a) ABC
 b) AC
 c) A
 d) B
- 9) The shortest word length in a defining relation is called as _____.
 a) Generator
 b) Alias
 c) Resolution
 d) All of these

- 10) In 3^3 factorial experiment with factors A and B, the interaction AB has _____ d.f.'s.
 a) 8
 b) 4
 c) 1
 d) depending on the experiment
- 11) For 2^k design the complete model would contain _____ effects.
 a) 2^{k-1}
 b) 2^k
 c) $2^k - 1$
 d) infinitely many
- 12) Consider the two statements
 P: A connected design always orthogonal,
 Q: An orthogonal design is always connected.
 Which of the statement is true?
 a) Only P
 b) Only Q
 c) Both P and Q
 d) Neither P or Q
- 13) For error degrees of freedom to be non zero in 2^k experiment, we have _____ replicates.
 a) 1
 b) At least 2
 c) Both a and b
 d) None of these
- 14) If there are four factors each at two levels and are conducted in two replications, then error degrees of freedom are _____.
 a) 0
 b) 13
 c) 12
 d) 16

Q.2 A) Answer the following (Any four) 08

- 1) Define BIBD and Symmetric BIBD.
- 2) Write one way ANOVA model with its assumptions.
- 3) Define interaction effect and its graphical representation in factorial design.
- 4) Shortly discuss the 3^2 factorial experiments with 2 replications.
- 5) Define generator and defining contrast subgroup in fractional factorial design.

B) Write Notes on (Any two) 06

- 1) Minimum Aberration Design
- 2) Total confounding
- 3) a) Resolution III b) Resolution IV in Design

Q.3 A) Answer the following (Any two) 08

- 1) Define ANOCOVA model in general. State ANOCOVA model in one way case.
- 2) Define Single replicate design. What is the need of single replicate design?
- 3) Describe 2^3 factorial experiment with 2 replications. Give suitable example of it.

B) Answer the following (Any one) 06

- 1) Write down the incidence matrix in case of BIBD with usual notations. Prove that in a BIBD number of blocks is greater than or equal to number of treatment.
- 2) Explain Main effect and interaction effect in a factorial experiments with special reference to 2^2 design. Give graphical representation of both effects.

Q.4 A) Answer the following (Any two) 10

- 1) Describe stepwise procedure of construction and analysis of 2^{4-1} experiment

- 2) Describe one way ANOVA model and obtain the least square estimates of its parameters
- 3) Define BIBD. What is symmetric BIBD? Prove that in case of BIBD $r(k - 1) = \lambda(v - 1)$.

B) Answer the following (Any one)**04**

- 1) Give Yates table for 2^3 factorial experiments. State the uses of Yates table.
- 2) Write down layout of 2^4 confounded design in two blocks with higher order interaction is confounded.

Q.5 Answer the following (Any two)**14**

- a) Discuss
 - 1) Randomization
 - 2) Replication
 - 3) Local control in Design of Experiments
- b) Explain full analysis of 2^3 factorial experiment with 3 replications. Give the Hypothesis under study, ANOVA table and test criterion.
- c) What are fractional factorial experiments? Give its uses with suitable example.



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M.Sc. (Part – II) (Semester – III) Examination, 2015
STATISTICS (Paper – XIII)
Planning and Analysis of Industrial Experiments

Day and Date : Monday, 20-4-2015
Time : 3.00 p.m. to 6.00 p.m.

Total Marks : 70

- Instructions:** 1) Attempt **five** questions.
2) Q. No. **1** and Q. No. **2** are **compulsory**.
3) Attempt **any three** from Q. No. **3** to Q. No. **7**.
4) Figures to the **right** indicate **full** marks.

1. A) Choose the correct alternative :

5

- 1) In the field experimentation, when experimental material is heterogeneous, we use _____
a) CRD b) RBD c) LSD d) None of these
- 2) The aliased defining relation of 2^{k-1} design is $I = ABCD$, then alias of A is _____
a) BCD b) ACD c) ABD d) ABC
- 3) The selection of p generator of 2^{k-p} fractional factorial design is in such a way that _____
a) it has lowest possible resolution
b) it has minimum aberration
c) it has highest possible resolution
d) it has maximum aberration
- 4) If all effects of same order are confounded with incomplete block difference, then it is said to be _____
a) complete confounding b) partial confounding
c) balanced confounding d) none of these
- 5) The objects which are to be compared in comparative experiment are called _____
a) blocks b) treatment
c) randomization d) all the above

P.T.O.



- B) Fill in the blanks : 5
- 1) In one-half fraction with $I = + ABC$ is called _____
 - 2) Replication reflects source of variability both _____ runs and _____ runs.
 - 3) Residual can be check by using formula $e_{ij} =$ _____
 - 4) Each contrast among k treatments has _____ degrees of freedom.
 - 5) The shortest word length is called _____
- C) State whether the following statements are **True** and **False** : 4
- 1) Factorial design is necessary when interaction may be present to avoid the misleading result.
 - 2) Fractional Factorial design reduces the number of levels of size.
 - 3) The procedure for moving sequentially along in the direction of maximization then it is said to be steepest descent.
 - 4) In randomization the treatments are allocated to the experimental units has with equal probability.
2. a) Answer the following : 6
- i) Define orthogonal array. Give its example.
 - ii) What is location and dispersion modeling ?
- b) Write short notes on the following : 8
- i) Total confounding.
 - ii) Control composite design.
3. a) Obtain a complete replicate with block size 8 for 2^5 factorial design having ABC and ADE interactions confounded simultaneously.
- b) Describe a 3^2 factorial experiment with factors A and B. Give complete procedure of analysis along with the ANOVA table. (7+7)



4. a) Explain Taguchi in design of experiment in terms of mode and layout of the experiment.
b) Find 2_{III}^{6-2} fraction having different aberrations. Use these fractions to state advantages of a fraction with less aberration over the other fractions. **(7+7)**
 5. a) Write a short note on Response Surface Methodology (RSM). What are response surface designs ? Write a note on design for fitting the first order modes.
b) Explain main effects and interactions in a factorial experiment with special reference to a 2^2 design. Give graphical representation to interaction in it. **(7+7)**
 6. a) Explain the one quarter fraction of 2^k experiment.
b) Describe the random effect model of one way classification. **(7+7)**
 7. a) Define resolution of a design. What is resolution III, IV and V design ?
b) Explain in brief about design of experiments. Also state the advantages and disadvantages of the same. **(7+7)**
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M.Sc. (Part – II) (Semester – III) Examination, 2015
STATISTICS (Paper – XIII)
Planning and Analysis of Industrial Experiments

Day and Date : Monday, 20-4-2015
Time : 3.00 p.m. to 6.00 p.m.

Total Marks : 70

- Instructions:** 1) Attempt **five** questions.
2) Q. No. **1** and Q. No. **2** are **compulsory**.
3) Attempt **any three** from Q. No. **3** to Q. No. **7**.
4) Figures to the **right** indicate **full** marks.

1. A) Choose the correct alternative :

5

- 1) In the field experimentation, when experimental material is heterogeneous, we use _____
a) CRD b) RBD c) LSD d) None of these
- 2) The aliased defining relation of 2^{k-1} design is $I = ABCD$, then alias of A is _____
a) BCD b) ACD c) ABD d) ABC
- 3) The selection of p generator of 2^{k-p} fractional factorial design is in such a way that _____
a) it has lowest possible resolution
b) it has minimum aberration
c) it has highest possible resolution
d) it has maximum aberration
- 4) If all effects of same order are confounded with incomplete block difference, then it is said to be _____
a) complete confounding b) partial confounding
c) balanced confounding d) none of these
- 5) The objects which are to be compared in comparative experiment are called _____
a) blocks b) treatment
c) randomization d) all the above

P.T.O.



- B) Fill in the blanks : 5
- 1) In one-half fraction with $I = + ABC$ is called _____
 - 2) Replication reflects source of variability both _____ runs and _____ runs.
 - 3) Residual can be check by using formula $e_{ij} =$ _____
 - 4) Each contrast among k treatments has _____ degrees of freedom.
 - 5) The shortest word length is called _____
- C) State whether the following statements are **True** and **False** : 4
- 1) Factorial design is necessary when interaction may be present to avoid the misleading result.
 - 2) Fractional Factorial design reduces the number of levels of size.
 - 3) The procedure for moving sequentially along in the direction of maximization then it is said to be steepest descent.
 - 4) In randomization the treatments are allocated to the experimental units has with equal probability.
2. a) Answer the following : 6
- i) Define orthogonal array. Give its example.
 - ii) What is location and dispersion modeling ?
- b) Write short notes on the following : 8
- i) Total confounding.
 - ii) Control composite design.
3. a) Obtain a complete replicate with block size 8 for 2^5 factorial design having ABC and ADE interactions confounded simultaneously.
- b) Describe a 3^2 factorial experiment with factors A and B. Give complete procedure of analysis along with the ANOVA table. (7+7)



4. a) Explain Taguchi in design of experiment in terms of mode and layout of the experiment.
b) Find 2_{III}^{6-2} fraction having different aberrations. Use these fractions to state advantages of a fraction with less aberration over the other fractions. **(7+7)**
 5. a) Write a short note on Response Surface Methodology (RSM). What are response surface designs ? Write a note on design for fitting the first order modes.
b) Explain main effects and interactions in a factorial experiment with special reference to a 2^2 design. Give graphical representation to interaction in it. **(7+7)**
 6. a) Explain the one quarter fraction of 2^k experiment.
b) Describe the random effect model of one way classification. **(7+7)**
 7. a) Define resolution of a design. What is resolution III, IV and V design ?
b) Explain in brief about design of experiments. Also state the advantages and disadvantages of the same. **(7+7)**
-

- 10) For 2^4 design the complete model would contain _____ effects.
 - a) 16
 - b) 14
 - c) 15
 - d) 32
- 11) BIBD is _____ orthogonal.
 - a) Always
 - b) Not
 - c) Sometimes
 - d) All of these
- 12) Preferably _____ interactions is chosen for confounding.
 - a) low order
 - b) middle order
 - c) higher order
 - d) none of these
- 13) Confounding is necessary to reduce _____.
 - a) Block size
 - b) No. of blocks
 - c) No. of factors
 - d) All of these
- 14) In the design matrix of Randomized block design all entries are _____.
 - a) One
 - b) zero
 - c) zero and one
 - d) any value between -1 and +1

Q.2 A) Answer the following questions. (Any Four) 08

- 1) Define main effect and interaction effect in factorial design.
- 2) Define Balancedness in design.
- 3) Write down two way ANOVA without interaction model with its assumptions.
- 4) Show that the Randomized Block Design is orthogonal design.
- 5) Write down aliases structure for 2^{3-1} design with generator as a higher order interaction.

B) Write short notes. (Any Two) 06

- 1) 2_{III}^{6-2} fractional factorial design
- 2) Complete confounding
- 3) i) Resolution IV
ii) Resolution V in Design

Q.3 A) Answer the following questions. (Any Two) 08

- 1) Define half fraction of 2^4 design with ABCD as g defining generator. Write a alias structure of it.
- 2) Discuss the use of confounding. State and describe the types of confounding.
- 3) Define BIBD. Obtain the determinant of incidence matrix in case of symmetric BIBD.

B) Answer the following questions. (Any One) 06

- 1) Describe two way ANOVA without interaction model with one observation per cell and obtain least square estimates of its parameter.
- 2) Define confounding. State its advantages and disadvantages.

Q.4 A) Answer the following questions. (Any Two) 10

- 1) Explain $\frac{1}{4}$ th fraction of 2^k experiment. Construct $\frac{1}{4}$ th fraction of 2^6 design with suitable example.
- 2) Discuss the two way ANOVA without interaction and ANOCOVA in one way case.
- 3) Describe the 2^3 factorial experiments. Explain the Yates procedure in case of 2^3 designs.

B) Answer the following questions. (Any One) 04

- 1) Write down layout of 2^4 confounded design in two blocks with higher order interaction is confounded.
- 2) Define
 - i) Principle Fraction
 - ii) Randomization in Design of Experiment

Q.5 Answer the following questions. (Any Two) 14

- a) Discuss the basic principles of Design of Experiments.
- b) What are fractional factorial experiments? Illustrate with $r = 1$ and $r = 2$ one example.
- c) State difference between analysis of 2^2 factorial experiments with $r = 2$. Explain full analysis of 2^2 factorial experiments for $r = 1$ and $r = 2$.



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M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – VII) (Old)
Linear Models and Design of Experiments

Day and Date : Saturday, 18-4-2015
Time : 11.00 a.m. to 2.00 p.m.

Total Marks : 70

- Instructions :** 1) Attempt **five** questions.
2) Q. No. **1** and Q. No. **2** are **compulsory**.
3) Attempt **any three** from Q. No. **3** to Q. No. **7**.
4) Figures to the **right** indicate **full** marks.

1. A) Select the correct alternative :

- 1) In general linear model, any linear function belonging to the error space and any BLUE are
 - a) Positively correlated
 - b) Uncorrelated
 - c) Negatively correlated
 - d) Correlated
- 2) The rank of the estimation space in one-way ANOVA model with N observations and k levels of treatment is
 - a) $N - k$
 - b) $k - 1$
 - c) k
 - d) $N - k - 1$
- 3) For a BIBD with usual notation, $\lambda(v-1) =$
 - a) $k(r-1)$
 - b) $k(r+1)$
 - c) $r(k+1)$
 - d) $r(k-1)$
- 4) In a connected block design with v treatments and b blocks, rank of C matrix is
 - a) $v-1$
 - b) $v+1$
 - c) v
 - d) $vb-1$
- 5) In a general linear model, the normal equation are
 - a) always consistent
 - b) not always consistent
 - c) always inconsistent
 - d) not always in consistent



B) Fill in the blanks :

- 1) In general linear model $y = X\beta + \epsilon$, a particular solution of the normal equations is _____
- 2) The rank of the estimation space in two-way ANOVA without interaction model with p rows and q columns and with one observation per cell is _____
- 3) A block design is _____ if and only if $CR^{-\delta} N = 0$.
- 4) The degrees of freedom of error SS in two-way ANOVA with interaction model with p rows and q columns and with $r > 1$ observation per cell is _____
- 5) The degrees of freedom of error SS in two-way without interaction ANOCOVA model with p rows, q columns, m observation per cell and m covariate is _____ **(1×5=5)**

C) State **True** or **False** :

- 1) In a general linear model, if S^{-} is g -inverse of $S = X'X$, its transpose is not in general g -inverse of S .
 - 2) $\mu + \alpha_i, i = 1, 2, \dots, k$, are estimable in one-way ANOVA model with k levels of treatment.
 - 3) In a general linear model $y = X\beta + \epsilon$, the quantity $XS^{-}X'$ is invariant under the choice of g -inverse of $S = X'X$.
 - 4) A balanced design is always connected. **(1×4=4)**
2. a) i) Show that any solution of normal equations minimizes residual sum of squares.
- ii) Prove or disprove that a connected design is always balanced. **(3+3)**
- b) Write short notes on the following :
- i) Estimation space.
 - ii) Tuckey's procedure of multiple comparisons. **(4+4)**



3. a) Show that in general linear model $y = X\beta + \epsilon$
- i) $H = H^2$
 - ii) $SH = H$
 - iii) $\text{rank}(H) = \text{trace}(H) = \text{rank}(S) = \text{rank}(X)$, where $S = X'X$, $H = S^- S$, S^- being g-inverse of S . (2+2+3)
- b) Prove that in a general linear model $y = X\beta + \epsilon$, the BLUE of every estimable linear parametric function is a linear function of the LHS of normal equations, and conversely, any linear function of the LHS of normal equations is the BLUE of its expected value. 7
4. a) Describe one-way ANOVA model and obtain the least square estimates of its parameters.
- b) Derive the test for testing the hypothesis of the equality of row effects in two-way ANOVA without interaction model with one observation per cell. (7+7)
5. a) Show that in general block design, adjusted treatment totals and block totals are uncorrelated.
- b) State and prove a necessary and sufficient condition for orthogonality of a connected block design. (7+7)
6. a) Describe ANOCOVA model in general and obtain the least square estimates of its parameters.
- b) Describe two-way with interaction ANOVA model with $r > 1$ observations per cell and obtain the least square estimates of its parameters. (7+7)
7. a) State and prove Gauss-Markoff theorem.
- b) Prove that in a BIBD, the number of blocks is greater than or equal to the number of treatments. (7+7)
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M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – VII) (Old)
Linear Models and Design of Experiments

Day and Date : Saturday, 18-4-2015
Time : 11.00 a.m. to 2.00 p.m.

Total Marks : 70

- Instructions :** 1) Attempt **five** questions.
2) Q. No. **1** and Q. No. **2** are **compulsory**.
3) Attempt **any three** from Q. No. **3** to Q. No. **7**.
4) Figures to the **right** indicate **full** marks.

1. A) Select the correct alternative :

- 1) In general linear model, any linear function belonging to the error space and any BLUE are
 - a) Positively correlated
 - b) Uncorrelated
 - c) Negatively correlated
 - d) Correlated
- 2) The rank of the estimation space in one-way ANOVA model with N observations and k levels of treatment is
 - a) $N - k$
 - b) $k - 1$
 - c) k
 - d) $N - k - 1$
- 3) For a BIBD with usual notation, $\lambda(v-1) =$
 - a) $k(r-1)$
 - b) $k(r+1)$
 - c) $r(k+1)$
 - d) $r(k-1)$
- 4) In a connected block design with v treatments and b blocks, rank of C matrix is
 - a) $v-1$
 - b) $v+1$
 - c) v
 - d) $vb-1$
- 5) In a general linear model, the normal equation are
 - a) always consistent
 - b) not always consistent
 - c) always inconsistent
 - d) not always in consistent



B) Fill in the blanks :

- 1) In general linear model $y = X\beta + \epsilon$, a particular solution of the normal equations is _____
- 2) The rank of the estimation space in two-way ANOVA without interaction model with p rows and q columns and with one observation per cell is _____
- 3) A block design is _____ if and only if $CR^{-\delta} N = 0$.
- 4) The degrees of freedom of error SS in two-way ANOVA with interaction model with p rows and q columns and with $r > 1$ observation per cell is _____
- 5) The degrees of freedom of error SS in two-way without interaction ANOCOVA model with p rows, q columns, m observation per cell and m covariate is _____ **(1×5=5)**

C) State **True** or **False** :

- 1) In a general linear model, if S^{-} is g -inverse of $S = X'X$, its transpose is not in general g -inverse of S .
 - 2) $\mu + \alpha_i, i = 1, 2, \dots, k$, are estimable in one-way ANOVA model with k levels of treatment.
 - 3) In a general linear model $y = X\beta + \epsilon$, the quantity $XS^{-}X'$ is invariant under the choice of g -inverse of $S = X'X$.
 - 4) A balanced design is always connected. **(1×4=4)**
2. a) i) Show that any solution of normal equations minimizes residual sum of squares.
- ii) Prove or disprove that a connected design is always balanced. **(3+3)**
- b) Write short notes on the following :
- i) Estimation space.
 - ii) Tuckey's procedure of multiple comparisons. **(4+4)**



3. a) Show that in general linear model $y = X\beta + \epsilon$
- i) $H = H^2$
 - ii) $SH = H$
 - iii) $\text{rank}(H) = \text{trace}(H) = \text{rank}(S) = \text{rank}(X)$, where $S = X'X$, $H = S^- S$, S^- being g-inverse of S . **(2+2+3)**
- b) Prove that in a general linear model $y = X\beta + \epsilon$, the BLUE of every estimable linear parametric function is a linear function of the LHS of normal equations, and conversely, any linear function of the LHS of normal equations is the BLUE of its expected value. **7**
4. a) Describe one-way ANOVA model and obtain the least square estimates of its parameters.
- b) Derive the test for testing the hypothesis of the equality of row effects in two-way ANOVA without interaction model with one observation per cell. **(7+7)**
5. a) Show that in general block design, adjusted treatment totals and block totals are uncorrelated.
- b) State and prove a necessary and sufficient condition for orthogonality of a connected block design. **(7+7)**
6. a) Describe ANOCOVA model in general and obtain the least square estimates of its parameters.
- b) Describe two-way with interaction ANOVA model with $r > 1$ observations per cell and obtain the least square estimates of its parameters. **(7+7)**
7. a) State and prove Gauss-Markoff theorem.
- b) Prove that in a BIBD, the number of blocks is greater than or equal to the number of treatments. **(7+7)**
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M.Sc. (Part – II) (Semester – III) Examination, 2015
STATISTICS (Paper – XIII)
Planning and Analysis of Industrial Experiments

Day and Date : Monday, 20-4-2015
Time : 3.00 p.m. to 6.00 p.m.

Total Marks : 70

- Instructions:** 1) Attempt **five** questions.
2) Q. No. **1** and Q. No. **2** are **compulsory**.
3) Attempt **any three** from Q. No. **3** to Q. No. **7**.
4) Figures to the **right** indicate **full** marks.

1. A) Choose the correct alternative :

5

- 1) In the field experimentation, when experimental material is heterogeneous, we use _____
a) CRD b) RBD c) LSD d) None of these
- 2) The aliased defining relation of 2^{k-1} design is $I = ABCD$, then alias of A is _____
a) BCD b) ACD c) ABD d) ABC
- 3) The selection of p generator of 2^{k-p} fractional factorial design is in such a way that _____
a) it has lowest possible resolution
b) it has minimum aberration
c) it has highest possible resolution
d) it has maximum aberration
- 4) If all effects of same order are confounded with incomplete block difference, then it is said to be _____
a) complete confounding b) partial confounding
c) balanced confounding d) none of these
- 5) The objects which are to be compared in comparative experiment are called _____
a) blocks b) treatment
c) randomization d) all the above

P.T.O.



- B) Fill in the blanks : 5
- 1) In one-half fraction with $I = + ABC$ is called _____
 - 2) Replication reflects source of variability both _____ runs and _____ runs.
 - 3) Residual can be check by using formula $e_{ij} =$ _____
 - 4) Each contrast among k treatments has _____ degrees of freedom.
 - 5) The shortest word length is called _____
- C) State whether the following statements are **True** and **False** : 4
- 1) Factorial design is necessary when interaction may be present to avoid the misleading result.
 - 2) Fractional Factorial design reduces the number of levels of size.
 - 3) The procedure for moving sequentially along in the direction of maximization then it is said to be steepest descent.
 - 4) In randomization the treatments are allocated to the experimental units has with equal probability.
2. a) Answer the following : 6
- i) Define orthogonal array. Give its example.
 - ii) What is location and dispersion modeling ?
- b) Write short notes on the following : 8
- i) Total confounding.
 - ii) Control composite design.
3. a) Obtain a complete replicate with block size 8 for 2^5 factorial design having ABC and ADE interactions confounded simultaneously.
- b) Describe a 3^2 factorial experiment with factors A and B. Give complete procedure of analysis along with the ANOVA table. (7+7)



4. a) Explain Taguchi in design of experiment in terms of mode and layout of the experiment.
b) Find 2_{III}^{6-2} fraction having different aberrations. Use these fractions to state advantages of a fraction with less aberration over the other fractions. **(7+7)**
 5. a) Write a short note on Response Surface Methodology (RSM). What are response surface designs ? Write a note on design for fitting the first order modes.
b) Explain main effects and interactions in a factorial experiment with special reference to a 2^2 design. Give graphical representation to interaction in it. **(7+7)**
 6. a) Explain the one quarter fraction of 2^k experiment.
b) Describe the random effect model of one way classification. **(7+7)**
 7. a) Define resolution of a design. What is resolution III, IV and V design ?
b) Explain in brief about design of experiments. Also state the advantages and disadvantages of the same. **(7+7)**
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M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – VII) (Old)
Linear Models and Design of Experiments

Day and Date : Saturday, 18-4-2015
Time : 11.00 a.m. to 2.00 p.m.

Total Marks : 70

- Instructions :** 1) Attempt **five** questions.
2) Q. No. **1** and Q. No. **2** are **compulsory**.
3) Attempt **any three** from Q. No. **3** to Q. No. **7**.
4) Figures to the **right** indicate **full** marks.

1. A) Select the correct alternative :

- 1) In general linear model, any linear function belonging to the error space and any BLUE are
 - a) Positively correlated
 - b) Uncorrelated
 - c) Negatively correlated
 - d) Correlated
- 2) The rank of the estimation space in one-way ANOVA model with N observations and k levels of treatment is
 - a) $N - k$
 - b) $k - 1$
 - c) k
 - d) $N - k - 1$
- 3) For a BIBD with usual notation, $\lambda(v-1) =$
 - a) $k(r-1)$
 - b) $k(r+1)$
 - c) $r(k+1)$
 - d) $r(k-1)$
- 4) In a connected block design with v treatments and b blocks, rank of C matrix is
 - a) $v-1$
 - b) $v+1$
 - c) v
 - d) $vb-1$
- 5) In a general linear model, the normal equation are
 - a) always consistent
 - b) not always consistent
 - c) always inconsistent
 - d) not always in consistent



B) Fill in the blanks :

- 1) In general linear model $y = X\beta + \epsilon$, a particular solution of the normal equations is _____
- 2) The rank of the estimation space in two-way ANOVA without interaction model with p rows and q columns and with one observation per cell is _____
- 3) A block design is _____ if and only if $CR^{-\delta} N = 0$.
- 4) The degrees of freedom of error SS in two-way ANOVA with interaction model with p rows and q columns and with $r > 1$ observation per cell is _____
- 5) The degrees of freedom of error SS in two-way without interaction ANOCOVA model with p rows, q columns, m observation per cell and m covariate is _____ **(1×5=5)**

C) State **True** or **False** :

- 1) In a general linear model, if S^{-} is g -inverse of $S = X'X$, its transpose is not in general g -inverse of S .
 - 2) $\mu + \alpha_i, i = 1, 2, \dots, k$, are estimable in one-way ANOVA model with k levels of treatment.
 - 3) In a general linear model $y = X\beta + \epsilon$, the quantity $XS^{-}X'$ is invariant under the choice of g -inverse of $S = X'X$.
 - 4) A balanced design is always connected. **(1×4=4)**
2. a) i) Show that any solution of normal equations minimizes residual sum of squares.
- ii) Prove or disprove that a connected design is always balanced. **(3+3)**
- b) Write short notes on the following :
- i) Estimation space.
 - ii) Tuckey's procedure of multiple comparisons. **(4+4)**



3. a) Show that in general linear model $y = X\beta + \epsilon$
- i) $H = H^2$
 - ii) $SH = H$
 - iii) $\text{rank}(H) = \text{trace}(H) = \text{rank}(S) = \text{rank}(X)$, where $S = X'X$, $H = S^{-}S$, S^{-} being g-inverse of S . (2+2+3)
- b) Prove that in a general linear model $y = X\beta + \epsilon$, the BLUE of every estimable linear parametric function is a linear function of the LHS of normal equations, and conversely, any linear function of the LHS of normal equations is the BLUE of its expected value. 7
4. a) Describe one-way ANOVA model and obtain the least square estimates of its parameters.
- b) Derive the test for testing the hypothesis of the equality of row effects in two-way ANOVA without interaction model with one observation per cell. (7+7)
5. a) Show that in general block design, adjusted treatment totals and block totals are uncorrelated.
- b) State and prove a necessary and sufficient condition for orthogonality of a connected block design. (7+7)
6. a) Describe ANOCOVA model in general and obtain the least square estimates of its parameters.
- b) Describe two-way with interaction ANOVA model with $r > 1$ observations per cell and obtain the least square estimates of its parameters. (7+7)
7. a) State and prove Gauss-Markoff theorem.
- b) Prove that in a BIBD, the number of blocks is greater than or equal to the number of treatments. (7+7)
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M.Sc. (Part – I) (Semester – II) Examination, 2014
STATISTICS (Paper – VII)
Linear Models and Design of Experiments

Day and Date : Thursday, 24-4-2014
Time : 11.00 a.m. to 2.00 p.m.

Max. Marks : 70

- Instructions:** 1) Attempt **five** questions.
2) Q. No. (1) and Q. No. (2) are **compulsory**.
3) Attempt **any three** from Q. No. (3) to Q. No. (7).
4) Figures to the **right** indicate **full** marks.

1. A) Choose correct alternative :

5

- 1) Suppose $\hat{\beta}_1$ and $\hat{\beta}_2$ are two different solutions of normal equations $X'X\hat{\beta} = X'Y$ then which of the following statement is true ?
- a) $X'X\hat{\beta}_1 = X'X\hat{\beta}_2$ b) $\hat{\beta}_1 = \hat{\beta}_2$
c) $V(\hat{\beta}_1) = V(\hat{\beta}_2)$ d) $E(\hat{\beta}_1) = E(\hat{\beta}_2)$
- 2) The degrees of freedom (d.f.) associated with error sum of squares for two-way classification without with one observation per cell when factor A is not at 'a' levels and factor B at 'b' are _____
- a) $(a - 1)(b - 1)$ b) $b(a - 1)$
c) $ab - 1$ d) $a(b - 1)$
- 3) If $X_1, X_2, X_3, \dots, X_n$ are iid then $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ and $\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2$ are _____
- a) iid b) independently distributed
c) identically distributed d) not iid



- 4) Assume that $X_i \sim N(0, 1)$, $i = 1, 2, \dots$ and independently distributed then the quadratic form $X'AX$ is distributed as _____
- Chi-square with d.f. equal to $R(A)$
 - Chi-square with d.f. equal to $R(A)$ if A is idempotent
 - Chi-square with d.f. equal to $\text{Rank}(A^2)$
 - Chi-square with d.f. equal to $\text{trace}(A)$
- 5) The LSE of β in the full rank linear model $Y = \alpha + \beta X + \varepsilon$ is _____
- inconsistent
 - biased
 - unbiased
 - none of these

B) Fill in the blanks :

5

- In the Gauss-Markov model $(Y_{n \times 1}, A\theta, \sigma^2 I_n)$, with $\text{rank}(A) = m$, $E(\text{RSS})$ is equal to _____
- In Gauss-Markov model, the sampling distribution of the test statistic for testing a linear hypothesis is _____ distribution.
- In the Gauss-Markov model, an unbiased estimated of the error variance is _____
- If $l'\theta$ is an estimable linear parametric function in a Gauss-Markov model $(Y, A\theta, \sigma^2 I_n)$ with $l'\hat{\theta}$, $\text{Cov}(l'\hat{\theta}, Y - A\hat{\theta}) =$ _____
- The error space and estimation space are _____ of each other.

C) State whether the following statement are **true (T)** or **false (F)** :

4

- If columns of X matrix are mutually orthogonal then all linear parametric functions are estimable.
- In two way classification with interaction and with one observation per cell, an estimate of σ^2 is always available.



3) The assumption of normality is required for testing hypothesis regarding the parameter in the usual linear model.

4) If $\underline{X} \sim N(0, \sigma^2 I)$ $\underline{X}'A\underline{X}$ and $\underline{X}'B\underline{X}$ are independently distributed then $AB = 0$

2. a) Consider the following two full rank models :

i) $Y_i = \alpha + \beta (X_i - \bar{X}) + \varepsilon_i, i = 1, 2, \dots, n$

ii) $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i, i = 1, 2, \dots, n.$

1) Obtain LSE of the parameters of both models.

2) Show that the estimates $\hat{Y}_i, i = 1, 2, \dots, n$ obtained from both the models are same.

b) Write short notes on the following :

1) Inter block analysis

2) Tukey's test for additivity. **(6+8)**

3. a) State and prove Gauss-Markov theorem.

b) Suppose $E(Y_1) = E(Y_2) = \theta$ and $V(Y_1) = 5\sigma^2, Cov(Y_1, Y_2) = \sigma^2$ and $V(Y_2) = 2\sigma^2$. Obtain BLUE of θ . **(8+6)**

4. a) Explain the following terms :

i) Error space

ii) Estimable parametric function

iii) Estimation space.

b) Explain the one-way ANOVA non full rank model. Write this model in matrix form. Examine which parametric functions are estimable under this model.

Obtain the BLUE $\hat{\psi}$ if exists, of $\psi = \sum a_i \alpha_i$, where a_i are known constants and $\sum a_i = 0$. **(6+8)**



5. a) Obtain C-matrix of BIBD and show that all treatment contrasts are estimable. Obtain BLUE and its variance for treatment contrast.
- b) Write down ANOCOVA model with single covariate with assumptions. Obtain BLUE of contrast $\alpha_i - \alpha_u$ ($i \neq u$) and its variance, where α 's are the parameters involved in the model. **(8+6)**
6. a) Obtain a condition for a connected design to be orthogonal.
- b) Explain Scheffe's multiple comparison procedure. **(6+8)**
7. a) State general block design model. Obtain reduced normal equations for estimating treatment effects.
- b) Develop a test for testing no interaction in two way ANOVA model with $r > 1$ observations per cell. **(6+8)**
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