Seat No.	01163

M.Sc. (Part - I) (Semester - I) (CBCS) Examination, November - 2019

STATISTICS/APPLIED STATISTICS AND INFORMATICS

Estimation Theory (Paper - IV) (Revised)

Sub. Code : 74910/74977

Day and Date : Friday, 29 - 11 - 2019 **Total Marks : 80**

Time : 11.00 a.m. to 02.00 p.m.

Instructions :
1) **Question No. 1 is compulsory.**
2) **Attempt any four questions from question numbers 2 to 7.**
3) **Figures to the right indicate full marks.**

Q1) Answer the following : **[8 × 2 = 16]**

a) Define minimal sufficient statistics. Let $X_1, X_2, \ldots, X_n$ be a random sample from $b(1, p)$ distribution. Obtain a minimal sufficient statistics for p.

b) Define power series distribution family. Give an example.

c) State a necessary and sufficient condition for an unbiased estimator to be MVBUE.

d) Define pitman family. Give an example.

e) Define MLE.

f) Define degree of an estimable parameter and kernel.

g) Define loss function and Bayes risk.

h) Define Bayes rule and state Bayes estimator under squared error loss function.

Q2) a) State and prove Basu's theorem.

b) Define complete family. Show that $\{U(0, \theta),\ \theta > 0\}$ is a complete family.

[8 + 8 = 16]

Q3) a) State and prove Rao-Blackwell theorem.

b) Show that the UMVUE, if exists, is unique.

[8 + 8 = 16]

Q4) a) State and prove Chapman-Robbins-Kiefer inequality.

b) Describe Fisher's scoring method of obtaining an MLE.

[8 + 8 = 16]

Q5) a) Let $X_1, X_2, ..., X_n$ be a random sample from $N(\theta, 1)$, $\theta \in R$, distribution. The prior distribution of θ is $N(0, 1)$. Find the Bayes estimator of θ under absolute error loss function.

b) Describe method of moments estimators. Let $X_1, X_2, ..., X_n$ be a sample from $N(\mu, \sigma^2)$ distribution. Obtain the method of moments estimator of (μ, σ^2).

[8 + 8 = 16]

Q6) a) Show that if T_1 and T_2 are two sufficient statistics, then T_1 is a function of T_2.

b) State and prove Lehmann-Scheffe theorem.

c) Let $X_1, X_2, ..., X_n$ be a random sample from $U\left[\theta-\frac{1}{2}, \theta+\frac{1}{2}\right]$ distribution. Obtain an MLE of θ.

d) Let $X \sim P(\lambda)$. $L(\lambda, d(x)) = (\lambda - d(x))^2$. The prior distribution of λ is $G(\alpha, \beta)$. Calculate the Bayes risk for $d(x) = x$.

[4 × 4 = 16]

Q7) Write short notes on the following. [4 × 4 = 16]

a) Curved exponential family

b) The regularity conditions of CR inequality

c) Method of minimum chi-square

d) U-statistic

B - 815

Total No. of Pages : 3

Seat No. 2002

M.Sc. (Part - I) (Semester - I) (CBCS) Examination, November - 2015

APPLIED STATISTICS AND INFORMATICS (Paper - IV)

Statistical Inference

Sub. Code : 61058

F-1 DEC 2015

Day and Date : Monday, 02 - 11 - 2015 Total Marks : 80

Time : 10.30 a.m. to 01.30 p.m.

Instructions : 1) Question No. 1 is compulsory.

2) Attempt any four questions from question numbers 2 to 7.

3) Figures to the right indicate full marks.

Q1) Answer the following : [16 × 1 = 16]

a) Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(0, \theta)$. What is a minimal sufficient statistic for θ?

b) Let $X \sim U(0, \theta)$; $\theta > 0$. What is the UMVUE of θ?

c) Define sufficient statistic.

d) Give an example of a two-parameter family of distributions that is not an exponential family.

e) Define Power Series distribution.

f) What is likelihood function?

g) Let X_1, X_2 be a random sample from pdf $f_\theta(x) = \theta/x^2$; $x > 0$, $\theta > 0$; 0; otherwise. What is the MLE of θ?

h) Define ancillary statistic.

i) Define hypothesis.

j) What is critical region?

k) What is size of a test?

l) Define a most powerful test.

m) Give a reasonable test for testing the null hypothesis $H_0\ \theta \le \theta_0$ against $H_1\ \theta > \theta_0$ based on a sample of size n for b(1, θ) family of distributions.

n) Give an example of a hypothesis testing problem for which uniformly most powerful test does not exist.

o) Define likelihood ratio test.

p) Define power function of a test.

Q2) a) Let $X_1, X_2, ..., X_n$ be a random sample from pdf

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(\log x - \mu)^2}; x > 0.$$ Obtain sufficient statistic for

i) μ when σ^2 is known,

ii) σ^2 when μ is known.

b) State and prove Basu's theorem. Give one application of Basu's theorem.

[8+8]

Q3) a) State and prove Lehmann-Scheffe theorem. Use it to obtain UMVUE of 1/θ based on a random sample of size n from U(0, θ) distribution.

b) Describe method of scoring and its application to estimation in multinomial distribution.

[8+8]

Q4) a) State and prove Neyman-Pearson lemma.

b) Obtain size α test for testing $H_0 : \beta = 1$ against $H_1 : \beta = \beta_1\ (> 1)$, based on a sample of size 1 from $f(x, \beta) = \beta x^{\beta-1}$, $0 < x < 1$; $= 0$, *otherwise*.

[8+8]

Q5) a) Define monotone likelihood ratio (MLR). Show that U(0, θ) has MLR in $X_{(n)}$.

b) Obtain the likelihood ratio test for testing $H_0 : p = p_0$ against $H_1 : p \neq P_0$, based on a sample of size 1 from b(n, p) distribution.

[8+8]

Q6) a) Show that the family of Binomial distributions is complete.

b) Show that not every function of a sufficient statistic is sufficient.

c) Show that C(1, 0) does not have monotone likelihood ratio.

d) Discuss existence and nonexistence of UMP tests.

[4×4]

Q7) Write short notes on the following : [4×4]

a) Maximum likelihood estimator

b) ~~Completeness and bounded completeness~~

c) Simple and composite hypotheses

d) Errors in hypothesis testing problem

Total No. of Pages : 3

Seat No. 01957

M.Sc. (Part - I) (Semester - I) (CBCS) Examination, November - 2014

APPLIED STATISTICS AND INFORMATION (Paper - IV)

Statistical Inference

Sub. Code : 61058

Day and Date : Monday, 17 - 11 - 2014

Total Marks : 80

Time : 10.30 a.m. to 1.30 p.m.

Instructions :
1) Q.1. is compulsory.
2) Attempt any 4 questions from Q. 2 - Q.7.
3) Figures to the right indicate marks.

Q1) Answer the following sub questions. [16]

a) Define a sufficient statistic and give an example for the same.

b) If $T(\underline{X})$ is sufficient for θ, then what is sufficient for $\log \theta$? Justify your answer

c) Define minimal sufficient statistic.

d) Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from Poisson distribution with mean λ. Obtain an unbiased estimator for λ^2.

e) Define minimum variance unbiased estimator. MVUE

f) State characterizing property of UMVUE.

g) State invariance property of maximum likelihood estimator (MLE).

h) Is MLE unique? Justify your answer.

i) Define simple and composite hypothesis.

j) Give an example of a randomized test.

k) Give an example of a test having power 1.

l) Is most powerful (MP) test unique? Justify your answer.

m) Define monotone likelihood ratio (MLR) property.

n) State relation between MP and UMP tests.

o) Define likelihood ratio test.

p) Comment on 'Likelihood ratio statistic lies between 0 and 1'.

Q2) a) State and prove factorization theorem for discrete case.

b) Define likelihood equivalence and give an example of likelihood equivalent sample points. Based on a random sample of size n from B(1, θ), obtain minimal sufficient statistics for θ.

[8 + 8]

Q3) a) Define completeness property. Prove that N (θ, 1) family is complete.

b) State and prove Basu's theorem and give an example of the same.

[8 + 8]

Q4) a) State and prove Rao-Blackwell theorem and give an application of it.

b) Based on a random sample of size n from U(0, θ). Obtain UMVUE for θ.

[8 + 8]

Q5) a) Let $X_1, X_2, \ldots, X_n$ be a random sample from U ($-\theta, \theta$). Obtain MLE for θ.

b) Describe method of scoring and illustrate it with suitable example.

[8 + 8]

Q6) a) Based on a random sample of size on $N(\theta, 1)$, obtain MP test of size α for testing $H_0: \theta = 0$ against $H_1: \theta = 1$.

b) Let ϕ be a MP test of size α for testing H_0 against H_1. Prove that $1-\phi$ is also a MP test for an appropriate problem.

c) Let H_0: $X \sim f_0$ against H_1: $X \sim f_1$, where f_0 and f_1 are as follows.

x	0	1	2	3
$f_0(x)$	0.25	0.25	0.25	0.25
$f_1(x)$	0.1	0.2	0.3	0.4

i) Obtain MP test of size 0.1 for testing H_0 against H_1.

ii) Obtain power of the test $\phi(x)$, where $\phi(x) = 1$ if $x = 0$ or 3 and $\phi(x) = 0$ otherwise.

[5 + 5 + 6]

Q7) Write short notes on the following.

a) UMP test for one sided alternative in case of one parameter exponential family.

b) Non-existence of UMP test for two-sided alternative.

c) LRT for testing independence of two attributes.

d) Bounded completeness.

[4 × 4]

CBCS E – 1147

Total No. of Pages : 2

Seat No.	3822.

M.Sc. (Part - I) (Semester - I) Examination, Dec. - 2013
APPLIED STATISTICS AND INFORMATICS (Paper - IV)
Statistical Inference
Sub. Code : 61058

Day and Date : Monday, 02 - 12 - 2013   Total Marks : 80

Time : 10.30 a.m to 1.30 p.m.

Instructions : 1) Question No. 1 is compulsory.
2) Attempt any four questions from question No. 2 to 7
3) Figures to right indicate marks to the questions.

Q1) Answer the following : [16 × 1 = 16]

a) What do you mean by a sufficient statistic?

b) State Basu's theorem.

c) Give an example of MLE that is not unbiased. ✓

d) Define an ancillary statistic.

e) Give an example of family of distributions that does not belong to the exponential class.

f) Define unbiased estimator.

g) State invariance property of MLE.

h) What is a randomized test?

i) Explain level of significance.

j) Define most powerful test.

k) Give example of simple and composite hypotheses.

l) Define MLR property.

m) Define power series family.

n) Explain the probability of type I error.

o) Let $X_1, X_2, \ldots, X_n$ be random sample from $N(0, \sigma^2)$, σ^2 unknown. Examine whether $\bar{X}$ is ancillary statistic.

p) State necessary and sufficient condition for existance of UMVUE.

P.T.O.

$T(x) = \psi(\theta) + \frac{\psi'(\theta)}{I(\theta)} \quad \frac{\partial \log f}{\partial \theta}$

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Q2) a) State and prove Neyman factorization theorem in case of discrete distribution. [8]

b) Let $X_1, X_2, \ldots, X_n$ be i.i.d. observations from $U(0, \theta)$ distribution. Use conditional definition to show $X_{(n)}$ is sufficient statistic for θ but $X_{(1)}$ is not sufficient statistic. [8]

Q3) a) Describe the method of maximum likelihood for estimating an unknown parameter. [8]

b) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\theta, \theta)$, distribution. $0 < \theta < \infty$. Obtain MLE of θ. [8]

Q4) a) Show that for a family having MLR property, there exists UMP test for testing one sided hypothesis against one sided alternative. [8]

b) Given a random sample of size n from $B(1, \theta)$ distribution, find UMP level α test of $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$. Derive power function $\beta(\theta)$ of the test. [8]

Q5) a) Define likelihood ratio test procedure for testing $H_0 : \theta \in \Theta_0$ against $H_1 : \theta \in \Theta_1$. Find LRT to test $H_0 : P = \frac{1}{2}$ against $H_1 : P \neq \frac{1}{2}$ based on sample of size one from $B(n, p)$ distribution. [8]

b) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N(\theta, \sigma^2)$, σ^2 unknown. Derive LRT to test $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$. [8]

Q6) a) Obtain a minimal sufficient statistic for one parameter exponential family of distributions.

b) State and prove Rao-Blackwell theorem.

c) Let $X_1, X_2, \ldots, X_n$ be a random sample from p.m.f. $P(X = K) = \frac{1}{N}$,

$K = 1, 2, \ldots, N.$

$= 0$, otherwise.

find UMVUE of N.

d) Examine whether $U(0, \theta)$, $\theta > 0$ possesses MLR Property.

[4 × 4]

Q7) Write short notes on the following :

a) Method of scoring to obtain MLE.

b) Bounded completeness.

c) Sufficiency in power series distributions.

d) Non-regular family of distributions.

Seat No.	01163

M.Sc. (Part - I) (Semester - I) (CBCS) Examination, November - 2019

STATISTICS/APPLIED STATISTICS AND INFORMATICS

Estimation Theory (Paper - IV) (Revised)

Sub. Code : 74910/74977

Day and Date : Friday, 29 - 11 - 2019 **Total Marks : 80**

Time : 11.00 a.m. to 02.00 p.m.

Instructions :
1) **Question No. 1 is compulsory.**
2) **Attempt any four questions from question numbers 2 to 7.**
3) **Figures to the right indicate full marks.**

Q1) Answer the following : **[8 × 2 = 16]**

a) Define minimal sufficient statistics. Let $X_1, X_2, ..., X_n$ be a random sample from $b(1, p)$ distribution. Obtain a minimal sufficient statistics for p.

b) Define power series distribution family. Give an example.

c) State a necessary and sufficient condition for an unbiased estimator to be MVBUE.

d) Define pitman family. Give an example.

e) Define MLE.

f) Define degree of an estimable parameter and kernel.

g) Define loss function and Bayes risk.

h) Define Bayes rule and state Bayes estimator under squared error loss function.

Q2) a) State and prove Basu's theorem.

b) Define complete family. Show that $\{U(0, \theta), \ \theta > 0\}$ is a complete family.

[8 + 8 = 16]

Q3) a) State and prove Rao-Blackwell theorem.

b) Show that the UMVUE, if exists, is unique.

[8 + 8 = 16]

Q4) a) State and prove Chapman-Robbins-Kiefer inequality.

b) Describe Fisher's scoring method of obtaining an MLE.

[8 + 8 = 16]

Q5) a) Let $X_1, X_2, ..., X_n$ be a random sample from $N(\theta, 1)$, $\theta \in R$, distribution. The prior distribution of θ is $N(0, 1)$. Find the Bayes estimator of θ under absolute error loss function.

b) Describe method of moments estimators. Let $X_1, X_2, ..., X_n$ be a sample from $N(\mu, \sigma^2)$ distribution. Obtain the method of moments estimator of (μ, σ^2).

[8 + 8 = 16]

Q6) a) Show that if T_1 and T_2 are two sufficient statistics, then T_1 is a function of T_2.

b) State and prove Lehmann-Scheffe theorem.

c) Let $X_1, X_2, ..., X_n$ be a random sample from $U\left[\theta-\frac{1}{2}, \theta+\frac{1}{2}\right]$ distribution. Obtain an MLE of θ.

d) Let $X \sim P(\lambda)$. $L(\lambda, d(x)) = (\lambda - d(x))^2$. The prior distribution of λ is $G(\alpha, \beta)$. Calculate the Bayes risk for $d(x) = x$.

[4 × 4 = 16]

Q7) Write short notes on the following. [4 × 4 = 16]

a) Curved exponential family

b) The regularity conditions of CR inequality

c) Method of minimum chi-square

d) U-statistic

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Total No. of Pages :3

Seat No.	2100

M.Sc.(Part -I)(Semester -I)(CBCS)
Examination, March - 2016
APPLIED STATISTICS AND INFORMATICS
Statistical Inference(Paper - IV)
Sub. Code: 61058

Day and Date : Monday, 28 - 3 - 2016

Time :11.00 a.m. to 2.00 p.m.

Total Marks :80

Instructions :
1) Quesiton No.1 is compulsory.
2) Attempt any four questions from Question No.2 to 7.
3) Figures to the right indicate marks to the questions.

Q1) Answer the following: **[16×1]**

a) Define bias of an estimator

b) Define minimal sufficient statistic

c) If $f(x) = \begin{cases} e^{-(x-0)}, & x > \theta \\ o & otherwise \end{cases}$

Find whether $\overline{X}$ is an unbiased estimator for θ

d) Define complete family of distributions

e) *u* is an ancilliary statistic, *t* is complete sufficient statistic, then distribution of *t* and *u* are independent or not?

f) Define one parameter exponential family.

g) If $\hat{\theta}$ is MLE of θ, then what will be mle of u(θ), u is single valued function of θ

h) State Rao-Blackwell theorem.

P.T.O.

i) What is composite hypothesis.

j) Explain power of the test.

k) Describe unbiasedness property of a test.

l) Describe monotone likelihood ratio property.

m) Define power series distribution.

n) Define an UMP test.

o) Describe non randomized test.

P) State the application of Basu's theorem

Q2) a) Let $\{x_n\}$ be a sequence of iid $N(\mu, \sigma^2)$ r.v.'s. Show that s^2 is consistent for σ^2.

b) State and prove Fisher- Neyman factorization theorem is case of discrete random variables.

[8+8]

Q3) a) Let X_i, i = 1,2, ...,n be a random sample from a $N(\mu, \sigma^2)$ population. Show that $\bar{X}$ is sufficient for μ when σ is known.

b) Let X_1,----,X_n be iid r.v.'s having density $f_\theta(x) = \frac{1}{2}\exp(-(x-\theta))$. Find MLE of θ.

[8+8]

Q4) a) Let X_1, -----, X_n be a random sample from pdf $f_\theta(x) = \frac{\theta}{x^2}$ if $0 < \theta \leq x < \infty$. Find MP test of size α for testing $H_0: \theta = \theta_0$ vs $H_1: \theta = \theta_1 (> \theta_0)$. Find the power of the test.

b) Show that there does not exist UMP test for testing two-sided alternatives in a one-parameter exponential family.

[8+8]

Q5) a) Let $X_1, X_2, ----, X_n$ be a random sample from $N(\mu, \sigma^2)$, where both μ and σ^2 are unknown. Obtain likelihood ratio test for testing $H_0: \mu = \mu_0$ vs $H_1: \mu \neq \mu_0$.

b) Obtain an asymptotic distribution of a likelihood ratio test statistic. **[8+8]**

Q6) a) Show that complete sufficient statistic is always minimal sufficient.

b) State and prove Lehmann-scheffe theorem.

c) Describe the procedure of finding minimum sample size to achieve the desired strength for MP test.

d) Explain how Likelihood Ratio test is used for contingency table. **[4×4]**

Q7) Write short notes on the following: **[4×4]**

a) Application of method of scoring.

b) Invariance property of MLE.

c) Sufficiency in non regular family of distributions.

d) Bounded completeness.

Seat No.	1403

B - 794

Total No. of Pages : 3

M.Sc. (Part - I) (Semester - I) (CBCS) Examination, November - 2015

STATISTICS (Paper - IV)

Estimation Theory

Sub. Code : 59761

1 DEC 2015

Day and Date : Monday, 02 - 11 - 2015

Time : 10.30 a.m. to 01.30 p.m.

Total Marks : 80

Instructions :

1) Question No. 1 is compulsory.
2) Attempt any four questions from questions No. 2 to 7.
3) Figures to right indicate full marks.

Q1) Answer the following : [16 × 1 = 16]

a) State the Neyman's factorization theorem for continuous distribution.

b) Define complete sufficient statistic and give an example for same.

c) Let X1, X2 are i.i.d. P(λ) distribution, Show that X1 + 2X2 is not sufficient for λ.

d) Define Pitman's family. Give an example of same.

e) Prove are disprove Unbiased estimators are not unique.

f) Define MVUE of parameter θ. Give an example of same.

g) Find Fisher's information function for parameter θ if

$$f(x,\theta)=(1-\theta)\theta^{x}, x=0,1,2,....,0<\theta<1.$$

h) Define Likelihood function.

i) Show that (r + 1) / (X + r + 1) is an unbiased estimator of p if X ~ NB (r, p) distribution.

j) State MLE of θ based on sample of size n from

$$f(x,\theta)=\exp-(x-\theta), x>\theta, \theta>0.$$

k) What statistic is used in minimum chi square method to estimate the parameter.

P.T.O.

l) Let X_1, X_2 be a random sample from $f(x,\theta)=\theta/x^2, x>\theta, \theta \in R+$. Find MLE of θ.

m) Give an example of conjugate family.

n) Define non-informative prior.

o) State Cramer - Rao inequality.

p) What is difference between ancillary statistic and pivot.

Q2) a) State and prove Neyman's factorization theorem for discrete family of distribution. [8]

b) Let Xi (i = 1, 2, ..., n) be i.i.d. observations from distribution with p.m.f., $P(X=k) = p.(1-p)^{k-1}$, $k = 1,2,...,0 < p < 1$

Find sufficient statistic. [8]

Q3) a) State and Prove Lehmann- Scheffe theorem. [8]

b) Given a random sample of size n from Poisson distribution with mean λ, Obtain UMVUE of $(\lambda+1)e^{-\lambda}$. [8]

Q4) a) Define Maximum Likelihood Estimator (MLE). Suppose that n observations are taken from $N(\theta,1)$ Distribution, but it is only recorded that whether observation is positive or not and not an actual value. If m out of n observations are positive, find MLE of θ. [8]

b) Let Xi (i = 1,2,...,n) be i.i.d. $U(\theta,\theta+1)$ distribution. Obtain MLE of θ. Is it unique? Justify your answer. [8]

Q5) a) State and Prove Bhattacharya Bounds. [8]

b) Describe the method of minimum chi-square for estimating an unknown parameter. [8]

Q6) a) Show that if MLE exists, it is a function of sufficient Statistic.

b) Using Basu's theorem, Show that statistic $X_{(1)}$ and $\sum(X_i - X_{(1)})$ based on random sample of from $\exp(\mu, \sigma)$ are independent.

c) Let $X_1, X_2, .., X_n$ be a random sample of size n from $B(1, \theta), 0 < \theta < 1$ and prior distribution of θ is Be_1 (α, β). Find posterior distribution of θ.

[4 + 6 + 6]

Q7) Write the short notes on the following : [4 × 4 = 16]

a) Completeness and bounded completeness

b) Invariance property of MLE

c) Various types of priors

d) Posterior distribution

Cr. G – 685

Total No. of Pages : 3

Seat No.	

M.Sc. (Part - I) (Semester - I) Examination, 2013
STATISTICS
Estimation Theory (Paper - IV) (Credit System)
Sub. Code : 42324

Day and Date : Friday, 26 - 04 - 2013 **Total Marks : 80**

Time : 3.00 p.m. to 6.00 p.m.

Instructions :
1) Question No. 1 is compulsory.
2) Attempt any four questions from No. 2 to 7.
3) Figures to right indicate marks to the sub-question.

Q1) Attempt <u>any eight</u> sub-questions : [$8 \times 2 = 16$]

a) Obtain a sufficient statistic for the power series family of distributions.

b) Let X_1, X_2 ----, X_n be i i d N $(\theta, 1)$. Find an unbiased estimator of θ^2.

c) Give an ancillary statistic based on two random variables X_1, X_2 from N (μ, σ^2) with μ unknown and σ^2 known. Explain why your statistic is ancillary.

d) Prove or disprove : order statistics are always sufficient.

e) Give an example of a statistic that is not minimal sufficient. Justify your claim.

f) State invariance property of maximum likelihood estimator.

g) Illustrate the applicability of Bascis theorem.

h) Define the Fisher information function based on single observation and on nobservations from a distribution with pdf F (x, θ).

i) State Chapman-Robbins-Kiefer inequality.

j) Define an estimable function and Kernel with reference to non-parametric estimation.

Q2) a) Define sufficient statistic and minimal sufficient statistic. Explain the method of constructing minimal sufficient statistic.

b) Define completeness. Let X be a N $(\theta, 1)$ random variable show that the family of X is complete. [8 + 8]

Q3) a) Explain the term uniformly minimum variance unbiased estimator (UMVUE). Show that UMVUE is uncorrelated with the unbiased estimator of O.

b) Let $X_1, X_2, ---- X_n$ be a random sample of size n from a poisson distribution with mean λ. Obtain UMVUE for $e^{-\lambda}$. [8 + 8]

Q4) a) State and prove Cramer-Rao (C-R) inequality with necessary regularity conditions.

b) Derive C-R lower bounel for an unbiased estimator of

i) Poisson mean θ.

ii) $e^{-\theta}$, based on a random sample of size n from Poisson (θ) distribution. [8 + 8]

Q5) a) State and prove Rao-Black well theorem.

b) Let $X_1, X_2, ---- X_n$ be a random sample from pmf

$$P(X = K) - \frac{1}{N}, \quad K = 1, 2, ----, N$$

$= 0$, otherwise find UMVUE of N. [8 + 8]

Q6) a) Describe the methods of moment estimator and minimum chisquare estimator.

b) Explain the method of scoring for estimating the parameter θ for a multinomial distribution where cell probabilities are known functions of a single parameter θ. [8 + 8]

Q7) Write short notes on any four of the following : [4 × 4]

a) Neyman - Fisher theorem.

b) Bounded completeness.

c) Bhattacharya bound.

d) Lehmann - Scheffe theorem.

e) Maximum Likelihood estimator.

f) U statistic theorems for one sample and two samples.

⊠⊠⊠⊠

Impr. Cr-W – 850

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2011
(Credit System)
STATISTICS (Paper – IV)
Estimation Theory

Day and Date: Tuesday, 26-4-2011 Total Marks: 80

Time: 11.00 a.m. to 2.00 p.m.

Instructions : *a) Question No. 1 is compulsory.*

b) Attempt any 4 questions from questions No. 2 to 7.

c) Figures to the right indicate marks to the sub-question.

1. Attempt **any** eight of the following sub-questions :

a) Define a sufficient statistic. Comment on – 'A random sample is always sufficient'.

b) Give an example of a statistic which is sufficient and unbiased.

c) Comment on 'There are infinitely many sufficient statistics for the parameter of interest'.

d) Define ancillary statistic and give an example of the same.

e) Comment on 'Every minimal sufficient statistic is a sufficient statistic'.

f) Define estimability of a parameter. Give an example of a parametric function which is not estimable.

g) Show with suitable example that method of moments need not provide unbiased estimator always.

P.T.O.

h) Define maximum likelihood estimator (MLE). Give an example of an MLE which is not unbiased.

i) Define Fisher information contained in a single observation. Obtain the same for Bernoulli variate.

j) Based on a random sample of size n from $B(1, \theta)$, obtain MLE for θ^2. (2×8)

2. a) State factorization theorem for obtaining sufficient statistic. How ? Obtain sufficient statistic for θ, where $X_1, X_2, ..., X_n$ are iid $U(0, \theta)$.

b) Let $X_1, X_2, ..., X_n$ be iid Poisson r.v. with mean λ. Obtain minimal sufficient statistic for λ. Is it unique ? Is it complete ? Justify your answer. (8+8)

3. a) Let $X_1, X_2, ..., X_n$ be iid $N(0, \sigma^2)$. Check whether (i) $\sum X_i^2$ and (ii) $\sum X_i$ are sufficient for σ^2.

b) Let $X_1, X_2, ... X_n$ be a random sample from exponential distribution with location parameter θ and the scale parameter 1. Show that $X_{(1)}$ is a complete sufficient statistic and is independent of $\sum(X_i - X_{(1)})$. (8+8)

4. a) State and prove Lehmann-Scheffe theorem.

b) Based on a random sample from $N(\theta, \sigma^2)$, obtain UMVUE for (i) μ (ii) $\mu + \sigma^2$.

c) Based on a random sample of size n from Bernoulli $(1, \theta)$, obtain UMVUE for $\theta(1-\theta)$. (5+6+5)

5. a) State and prove Cramer-Rao inequality.

b) Based on a random sample of size n from exponential distribution with mean θ, obtain lower bound for the variance of an unbiased estimator for θ.

c) Comment on 'Every unbiased estimator attains Cramer-Rao lower bound for variance'. (8+5+3)

6. a) Let $X_1, X_2, \ldots, X_n$ be iid $U[\theta, \theta+1]$. Obtain MLE for θ. Is its unique unbiased MLE ? Justify your answer.

   b) Let $X_1, X_2, \ldots X_n$ be iid rvs having continuous distribution F. Obtain U-statistic for $P(X > a)$. Expression for variance of U-statistic. (8+8)

7. Write short notes on any four of the following :

   a) Bhattacharya bound

   b) Rao-Blackwell theorem and its applications

   c) Method of scoring

   d) Method of minimum chi-square

   e) Sufficiency for the parameter of one-parameter exponential distribution

   f) Relation between complete and boundedly complete statistic. (4×4)