Seat No. 01163

M.Sc. (Part - I) (Semester - I) (CBCS) Examination, November - 2019

STATISTICS/APPLIED STATISTICS AND INFORMATICS (Paper - II)

Linear Algebra (New)

Sub. Code : 74908/74975

Day and Date : Monday, 25 - 11 - 2019 **Total Marks : 80**

Time : 11.00 a.m. to 02.00 p.m.

Instructions :
1) **Question No. 1 is Compulsory.**
2) **Attempt any four questions from question numbers 2 to 7.**
3) **Figures to the right indicate full marks.**

Q1) Write answer of the following subquestions.

a) Show that vectors $x=\begin{pmatrix}1\\2\\3\end{pmatrix}$ and $y=\begin{pmatrix}2\\-2\\0\end{pmatrix}$ are linearly independent.

b) Define symmetric and skew-symmetric matrices.

c) If $A=[1,2,3]$ then fine a g-inverse of A and verify the same.

d) Write down the quadratic form corresponding to a matrix $A=\begin{bmatrix}1&4&6\\4&0&2\\6&2&5\end{bmatrix}$

e) Let λ be the characteristic root of non-singular matrix A. Show that $\frac{1}{\lambda}$ is characteristic root of A^{-1}.

f) Prove that if inverse of a matrix exists then it is unique.

g) Define basis and orthonormal basis.

h) Examine the equations $x+2y=5,\ 2x+4y=8$ for consistency.

[8 × 2]

P.T.O.

Q2) a) Define and illustrate giving one example each of the following terms.

i) Dimension of a vector space.

ii) Rank of a matrix

iii) idempotent matrix

iv) kro necker product of two matrices.

b) Let a_1, a_2 ----- a_n be a basis of a vector space. and b is a non-null vector. If $b = \sum_{i=1}^{n} \alpha_i a_i$ and any vector a_i for which $\alpha_i \neq 0$ is replaced by b then prove that the new collection of vectors is also a basis.

[8 + 8]

Q3) a) Define moore - penrose (mp) inverse. Show that each m×n matrix A, there exists one and only one n×m matrix A^+ (MP-inverse) with stating conditions.

b) Prove that the number of linearly independent solutions of system of equations $AX = 0$ is n-r, where r is the rank of matrix A. **[8+8]**

Q4) a) Define characteristic roots and vectors. find the characteristic roots and vectors of a matrix $A = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$, Hence find characteristic roots of matrix $B = A+3I$.

b) Explain :

i) Spectral decomposition of a real symmetric matrix A.

ii) Choleski decomposition of real symmetric positive definite matrix (A)

[8+8]

Q5) a) Define quadratic form and discuss definiteness of quadratic form with suitable example.

b) Prove that a necessary and sufficient condition for a quadratic form $X^T AX$ to be positive definite is that.

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1i} \\ a_{21} & a_{22} & & a_{2i} \\ \vdots & \vdots & \cdots & \vdots \\ a_{i1} & a_{i2} & & a_{ii} \end{vmatrix} > 0 \text{ for } i = 1, 2 \cdots n$$

[8+8]

Q6) a) Define the following and give one example each.

i) Permutation matrix

ii) Transpose of permutation matrix

iii) reducible matrix

iv) Primetive matrix

b) Find the value of λ and μ, the system of equations.

$x + y + z = 6$

$x + 2y + 3y = 10$

$x + 2y + \lambda z = \mu$

has i) a unique solution

ii) an infinite solution

iii) no solution

[8+8]

Q7) Write short notes on the following.

a) Inverse of a partitioned matrix.

b) vector space.

c) Extrema of a quadratic form

d) Cayley - Hamilton theorem

[4×4]

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