SLR-JS-373

Seat No.	

Set P

M.Sc. (Semester - II) (CBCS) Examination Oct/Nov-2019
Statistics
PROBABILITY THEORY

Day & Date: Monday, 04-11-2019 Max. Marks: 70
Time: 11:30 AM To 02:00 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. 14

1) A σ-field is closed under _________.
   a) complimentation b) countable union
   c) countable intersection d) all of these

2) For a sequence $\{A_n\}$ of sets, ________.
   a) $\overline{lim} A_n = \underline{lim}\, A_n$ b) $\overline{lim} A_n \subset \underline{lim}\, A_n$
   c) $\underline{lim}\, A_n \subset \overline{lim} A_n$ d) None of these

3) The smallest field of subsets of Ω contains ________ sets.
   a) 3 b) 4
   c) 2 d) cannot be predicted

4) The simple function is _________ linear combination of indicator of sets.
   a) finite b) arbitrary
   c) any d) none of these

5) If A and B are two subsets of Ω, then $P(A \cap B)$ _________.
   a) $= P(A) + P(B)$ b) $< P(A) + P(B)$
   c) $> P(A) + P(B)$ d) None of these

6) The characteristic function of a random variable X degenerate at c, equals _________.
   a) e^{itc} b) itc
   c) $i(c + t)$ d) e^{-tc}

7) A non-negative measure μ defined on subsets of Ω is said to be finite if and only if _________.
   a) $\mu(\Omega) = k < \infty$
   b) $\mu(\Omega) = 2$
   c) there are finitely many subsets of Ω
   d) all the these

8) Convergence in probability implies __________.
   a) convergence in distribution b) convergence in rth mean
   c) convergence in almost sure d) None of these

9) If A contains finite number of elements, then set A^C is called as ________.
   a) cofinite b) slightly finite
   c) nearly finite d) None of these

10) Expectation of non-negative random variable follows ________.
   a) linearity b) scale preserving
   c) both (a) and (b) d) None of these

11) If P(.) is a probability measure on (Ω, F) and if $P(A) = 1$, then A is _______.
    a) Φ b) Ω
    c) may or may not be Ω d) None of these

12) A r.v. X is integrable, if and only if __________.
    a) $\sin X$ is integrable b) X^2 is integrable
    c) $|X|$ is integrable d) None of these

13) If $X \geq 0$ a.s., then $E(X)$ _________.
    a) can be negative b) ≥ 0
    c) $= 0$ d) none of these

14) Which of the following is an elementary random variable?
    a) Bernoulli r.v. b) Geometric r.v.
    c) binomial r.v. d) None of these

Q.2 A) Answer the following questions. (Any Four) **08**

1) Define probability measure.
2) Define Lebesgue measure.
3) Define σ-field.
4) State Liaponove's Theorem on CLT
5) Define indicator function.

B) Write notes. (Any Two) **06**

1) Prove that collection of sets whose inverse images belong to a σ-field, is a also a σ-field.
2) Prove or disprove: Intersection of two fields is a field.
3) Discuss the construction of σ-field induced by r.v. X.

Q.3 A) Answer the following questions. (Any Two) **08**

1) Define conditional probability measure. Show that it is also a probability measure.
2) Prove or disprove: Mapping preserves all set relations.
3) Prove or disprove: Arbitrary union of fields is a field.

B) Answer the following questions. (Any One) **06**

1) Discuss limit superior and limit inferior of a sequence of sets. Find the same for sequence $\{A_n\}$, where $A_n = \left(0,\ 3 + \frac{(-1)^n}{n}\right), n \in N$
2) State and prove Fatou's lemma.

Q.4 A) Answer the following questions. (Any Two) **10**

1) State the constructive definition of arbitrary random variable using simple random variable. Justify.
2) Define convergence in probability and convergence in distribution. Prove or disprove: convergence in distribution implies convergence in probability.
3) Define expectation of simple random variable. If X and Y are simple random variables, prove the following:
   i) $E(X + Y) = E(X) + E(Y)$
   ii) $E(cX) = c\,E(X)$, where c is a real number
   iii) If $X > 0$ a.s., then $E(X) > 0$.

B) Answer the following questions. (Any One) **04**

1) Define the characteristic function of a r.v. and find the same for exponential distribution.
2) Prove any three properties of indicator function.

Q.5 Answer the following questions. (Any Two) **14**

- **a)** State and prove monotone convergence theorem.
- **b)** Prove that if $\{B_n\}$ converges to B, then $P(B_n)$ also converges to $P(B)$.
- **c)** Show that there are classes which are field but not σ-field.

SLR-BP – 482

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – VI) (Old)
Probability Theory

Day and Date : Thursday, 16-4-2015 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **(1)** and Q. No. **(2)** are **compulsory**.*
*3) Attempt **any three** from Q. No. **(3)** to Q. No. **(7)**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) Expectation of random variable $X = X^+ - X^-$ is said to exist if ___________

a) at least one of $E(X^+)$ or $E(X^-)$ is finite

b) both $E(X^+)$ and $E(X^-)$ are finite

c) both $E(X^+)$ and $E(X^-)$ are infinite

d) none of these

2) Which one of the following statement is correct ?

a) every field is a σ-field

b) union of fields is a field

c) intersection of fields is a field

d) $\{A, A^C\}$ is a field, where A is proper non-empty subset of Ω .

3) Which one of the following statement is correct ?

a) $X_n \xrightarrow{a.s.} X \Rightarrow X_n \xrightarrow{P} X$

b) $X_n \xrightarrow{L} X \Rightarrow X_n \xrightarrow{P} X$

c) $X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{a.s.} X$

d) $X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{r} X$

P.T.O.

4) The trivial field is ______________

a) $\{\phi\}$ b) $\{\Omega\}$

c) $\{\phi, \Omega\}$ d) none of these

5) Let X Poisson (λ) variate. Then characteristic function of X is ____________

a) $e^{-\lambda(1-e^{it})}$ b) $e^{\lambda(1-e^{-it})}$

c) $e^{-\lambda(e^{it}-1)}$ d) $e^{\lambda(e^{-it}-1)}$

B) Fill in the blanks : **5**

1) If $\phi_x(t)$ is real then X is ____________

2) A sequence of sets $\{A_n\}$ is said to be monotonic increasing if ____________

3) If $X_n \xrightarrow{r} X$ then $E|X_n|^r \to$ ____________

4) If A is finite set with n elements then power set of A contains __________ elements.

5) If P $(\cdot)$ is a probability measure then $P(A^C)$ = ____________

C) State whether the following statements are **TRUE** or **FALSE**. **4**

1) Almost sure convergence always implies convergence in probability.

2) Every field is a σ-field.

3) Mutual independence implies Pairwise independence.

4) Every field contains empty set ϕ.

2. a) Answer the following : **6**

i) Define field and σ-field.

ii) Prove that every σ-field is a field.

b) Write short notes on the following : **8**

i) Mutual and Pairwise independence.

ii) Strong law of large numbers.

3. a) If F_1 and F_2 are fields. Show that

   i) $F_1 \cap F_2$ is a field.

   ii) $F_1 \cup F_2$ is not a field.

   b) Find lim A_n if exist. $A_n = \left(1+\frac{1}{n}, 2+\frac{1}{n}\right)$ **(8+6)**

4. a) State and prove continuity property of probability measure.

   b) If $X_n \leq Y$ and Y is integrable then show that $E(\overline{\lim} X_n) \geq \overline{\lim} E(X_n)$. **(7+7)**

5. a) Prove that $X_n \xrightarrow{P} 0$ if and only if $E\left(\frac{|X_n|}{1+|X_n|}\right) \to 0$ as $n \to \infty$.

   b) Let $\{X_n\}$ be a sequence of random variables such that $X_n \xrightarrow{L} X$ and c be a constant. Show that

   i) $X_n + c \xrightarrow{L} X + c$

   ii) $c X_n \xrightarrow{L} cX, c \neq 0$. **(6+8)**

6. a) State Kolmogrov's three series criterion for almost sure convergence.

   b) Let $\{A_n\}$ be a sequence of events such that $\sum_{n=1}^{\infty} P(A_n) < \infty$. Show that

   $P(\overline{\lim} A_n) = 0$. **(6+8)**

7. a) Define characteristic function. Suppose X is B(n, p) random variable. Obtain characteristic function of X.

   b) State inversion formula and obtain the probability distribution of random variable corresponding to characteristic function $\phi_X(t) = \frac{1}{1+t^2}$. **(6+8)**

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Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – VI) (Old)
Probability Theory

Day and Date : Thursday, 16-4-2015 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions *: 1) Attempt* ***five*** *questions.*
2) Q. No. ***(1)*** *and Q. No.* ***(2)*** *are* ***compulsory****.*
3) Attempt ***any three*** *from Q. No.* ***(3)*** *to Q. No.* ***(7)****.*
4) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Choose the correct alternative : **5**

1) Expectation of random variable $X = X^+ - X^-$ is said to exist if ___________

a) at least one of $E(X^+)$ or $E(X^-)$ is finite

b) both $E(X^+)$ and $E(X^-)$ are finite

c) both $E(X^+)$ and $E(X^-)$ are infinite

d) none of these

2) Which one of the following statement is correct ?

a) every field is a σ-field

b) union of fields is a field

c) intersection of fields is a field

d) $\{A, A^C\}$ is a field, where A is proper non-empty subset of Ω .

3) Which one of the following statement is correct ?

a) $X_n \xrightarrow{a.s.} X \Rightarrow X_n \xrightarrow{P} X$

b) $X_n \xrightarrow{L} X \Rightarrow X_n \xrightarrow{P} X$

c) $X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{a.s.} X$

d) $X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{r} X$

4) The trivial field is ______________

a) $\{\phi\}$

b) $\{\Omega\}$

c) $\{\phi, \Omega\}$

d) none of these

5) Let X Poisson (λ) variate. Then characteristic function of X is ____________

a) $e^{-\lambda(1-e^{it})}$

b) $e^{\lambda(1-e^{-it})}$

c) $e^{-\lambda(e^{it}-1)}$

d) $e^{\lambda(e^{-it}-1)}$

B) Fill in the blanks : **5**

1) If $\phi_x(t)$ is real then X is ____________

2) A sequence of sets $\{A_n\}$ is said to be monotonic increasing if ____________

3) If $X_n \xrightarrow{r} X$ then $E|X_n|^r \rightarrow$ __________

4) If A is finite set with n elements then power set of A contains __________ elements.

5) If P $(\cdot)$ is a probability measure then $P(A^C) =$ ____________

C) State whether the following statements are **TRUE** or **FALSE**. **4**

1) Almost sure convergence always implies convergence in probability.

2) Every field is a σ-field.

3) Mutual independence implies Pairwise independence.

4) Every field contains empty set ϕ.

2. a) Answer the following : **6**

i) Define field and σ-field.

ii) Prove that every σ-field is a field.

b) Write short notes on the following : **8**

i) Mutual and Pairwise independence.

ii) Strong law of large numbers.

3. a) If F_1 and F_2 are fields. Show that

   i) $F_1 \cap F_2$ is a field.

   ii) $F_1 \cup F_2$ is not a field.

   b) Find lim A_n if exist. $A_n = \left(1+\frac{1}{n}, 2+\frac{1}{n}\right)$ **(8+6)**

4. a) State and prove continuity property of probability measure.

   b) If $X_n \leq Y$ and Y is integrable then show that $E(\overline{\lim} X_n) \geq \overline{\lim} E(X_n)$. **(7+7)**

5. a) Prove that $X_n \xrightarrow{P} 0$ if and only if $E\left(\frac{|X_n|}{1+|X_n|}\right) \to 0$ as $n \to \infty$.

   b) Let $\{X_n\}$ be a sequence of random variables such that $X_n \xrightarrow{L} X$ and c be a constant. Show that

   i) $X_n + c \xrightarrow{L} X + c$

   ii) $c X_n \xrightarrow{L} cX, c \neq 0$. **(6+8)**

6. a) State Kolmogrov's three series criterion for almost sure convergence.

   b) Let $\{A_n\}$ be a sequence of events such that $\sum_{n=1}^{\infty} P(A_n) < \infty$. Show that

   $P(\overline{\lim} A_n) = 0$. **(6+8)**

7. a) Define characteristic function. Suppose X is B(n, p) random variable. Obtain characteristic function of X.

   b) State inversion formula and obtain the probability distribution of random variable corresponding to characteristic function $\phi_X(t) = \frac{1}{1+t^2}$. **(6+8)**

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SLR-BP – 477

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – VI)
Probability Theory (New)

Day and Date : Thursday, 16-4-2015 Total Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. a) Choose the correct alternative. **5**

1) Let P(.) is a probability measure defined on (Ω, IF). Then $P(\Omega) = 1$ is ______ property of the measure.

a) Normed b) Non-negativity
c) Finite additivity d) Sigma additivity

2) If A_n is equal to A or B according as n is odd or even, the $\overline{\lim} A_n =$

a) 0 b) 1
c) $A \cap B$ d) $A \cup B$

3) Let X be a random variable defined on (Ω, IF) then

a) X^2 is a random variable b) $1 - X$ is a random variable
c) |X| is a random variable d) All the above

4) If X_1 and X_2 are independent random variables then $\phi_{x_1+x_2}(t) =$

a) $\phi x_1(t) + \phi x_2(t)$ b) $\phi x_1(t)\ \phi x_2(t)$
c) $\phi x_1(t)$ d) $\phi x_2(t)$

5) Which of the following are Borel sets of real line ?

a) Single point sets b) Open intervals of type (a, b)
c) Closed intervals of type [a, b] d) All the above

b) Fill in the blanks : **5**

1) If E(X) is finite then X said to be

2) A finite linear combination of indicators of sets is called ________ function.

3) The minimal σ-field induced by indicator function I_A is

4) The number of points in a set is called

5) A set A is called co-finite set if

c) State whether the following statements are **true** or **false**. **4**

1) The counting measure is a finite measure.

2) Mutual independence implies pairwise independence.

3) If Ω is the set of convergence then $\{X_n\}$ is said to be converge nowhere.

4) Mapping preserves all the set relations.

2. a) Answer the following : **6**

i) For a non-negative random variable X, prove that $E(X) = \int_0^\infty [1 - F(x)]dx$.

ii) Define lim inf and lim sup of sequence of sets $\{A_n\}$.

b) Write short notes on the following : **8**

i) Lebesgue measure.

ii) Indicator function.

3. a) Define monotone decreasing sequence of sets. Prove that if A_n is decreasing sequence of sets then A_n^c is increasing sequence.

b) Find lim inf and lim sup of following sequence of sets.

i) $A_n = \left(1 + \frac{1}{n}, 2 + \frac{1}{n}\right)$

ii) $A_n = \left[a - \frac{1}{n}, a\right]$ **(6+8)**

4. a) Define field. Examine for the class of finite or co-finite sets to be a field.

b) Define probability measure. State and prove monotone property of probability measure. **(7+7)**

5. a) If X and Y are simple random variables then prove that $E(X + Y) = E(X) + E(Y)$.

   b) Let E be an experiment having two outcomes 'success' S and 'failure' F respectively. Let $\Omega = \{S, F\}$ and $IF = \{\phi, S, F, \Omega\}$. Define $X(\omega) = \begin{cases} 1, & \text{if } \omega = S \\ 0, & \text{if } \omega = F \end{cases}$. Examine whether X is random variable with respect to IF. **(7+7)**

6. a) Define almost sure convergence. Prove that almost sure convergence implies convergence in probability.

   b) State Lindberg-Feller form of central limit theorem and deduce the Liapunov's theorem. **(7+7)**

7. a) Define characteristic function of random variable X. Suppose X is Poisson (λ) random variable. Obtain characteristic function of X.

   b) Find the distribution of random variable X when characteristic function is

      i) $\phi_X(t) = \dfrac{1}{1+t^2}$

      ii) $\phi_X(t) = e^{-|t|}$. **(6+8)**

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SLR-ES-371

Seat No.	

Set P

M.Sc. (Semester - II) (CBCS) Examination March/April-2019
Statistics
PROBABILITY THEORY

Day & Date: Saturday, 20-04-2019 Max. Marks: 70
Time: 12:00 PM To 02:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Choose Correct Alternative from the following. **14**

1) A field is closed under ______.
   a) complimentation b) finite union
   c) finite intersection d) all of these

2) For a sequence $\{A_n\}$ of sets, ______.
   a) $\overline{\lim} A_n = \underline{lim} A_n$ b) $\overline{\lim} A_n \subset \underline{lim} A_n$
   c) $\underline{lim} A_n \subset \overline{\lim} A_n$ d) None of these

3) The largest field of subsets of Ω is called as ______.
   a) Universal set b) Universal class
   c) Power set d) None of these

4) The elementary function is ______ linear combination of indicator of sets.
   a) finite b) arbitrary
   c) any d) none of these

5) If A and B are two subsets of Ω, then P(AUB) ______.
   a) = P(A) - P(B) b) < P(A) + P(B)
   c) > P(A)+P(B) d) None of these

6) If X is degenerate random variable at c, then characteristic function of X equals ______.
   a) e^{itc} b) itc
   c) i(c+t) d) e^{-tc}

7) Conditional probability measure is ______.
   a) non-negative b) normed
   c) countably additive d) all of these

8) Convergence almost sure implies ______.
   a) convergence in probability and convergence in distribution
   b) convergence in probability alone
   c) convergence in distribution alone
   d) convergence in probability, in distribution and in r^{th} mean

9) If A^C contains finite number of elements, then set A is called as ______.
   a) nearly finite b) slightly finite
   c) C-finite d) None of these

10) Expectation of simple random variable follows ______.
   a) linearity b) scale preserving
   c) both a and b d) none of these

11) Probability measure is ______.
   a) monotonic b) non-negative
   c) countably additive d) all of the above

12) Convergence in law is also called as ______.
   a) convergence in probability
   b) convergence in distribution
   c) convergence in r^{th} mean
   d) almost sure convergence

13) Which of the following is a simple random variable?
   a) Poisson r.v.
   b) Geometric r.v.
   c) discrete uniform r.v.
   d) none of these

14) A finite linear combination of indicators of sets is called _____ function.
   a) simple
   b) elementary
   c) arbitrary
   d) none of these

Q.2 A) Answer the following (Any four) **08**
1) Define measurable function and a measure.
2) Define Lebesgue-Stieltje's measure.
3) Define field.
4) State Lindeberg-Feller Theorems on CLT.
5) Define characteristic function of a random variable.

B) Write Short Notes(Any two) **06**
1) Prove that the probability measure is a monotonic measure. Also prove that $P(\Phi)=0$
2) Prove or disprove: Union of two fields is a field.
3) Discuss σ-field induced by r.v. X.

Q.3 A) Answer the following (Any two) **08**
1) Define mixture of two probability measures. Show that mixture is also a probability measure.
2) Prove that inverse mapping preserves all set relations.
3) Prove or disprove: Arbitrary intersection of fields is a field.

B) Answer the following (Any one) **06**
1) Prove that probability measure is a continuous measure.
2) State and prove Fatou's lemma.

Q.4 A) Answer the following (Any two) **10**
1) Prove that an arbitrary random variable can be expressed as a limit of sequence of simple random variables.
2) Define convergence in probability and convergence in distribution. Also prove that convergence in probability implies convergence in distribution.
3) Define expectation of simple random variable. If X and Y are simple random variables prove the following
   i) $E(X + Y) = E(X) + E(Y)$
   ii) $E(cX) = c\,E(X)$, where c is a real number
   iii) If $X > 0$ a.s., then $E(X) > 0$

B) Answer the following (Any one) **04**
1) Find the characteristic function for binomial distribution.
2) Prove any three properties of characteristic function.

Q.5 Answer the following (Any two) **14**
a) State and prove monotone convergence theorem.
b) Prove that inverse image of σ-field is also a σ-field.
c) Discuss Borel σ-field. Find the Borel sets.

SLR-MB – 618

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2016
STATISTICS (Paper – VI)
Probability Theory (Old CGPA)

Day and Date : Wednesday, 30-3-2016 Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **(1)** and Q. No. **(2)** are **compulsory**.*
*3) Attempt **any three** from Q. No. **(3)** to Q. No. **(7).***
*4) Figures to the **right** indicate **full** marks.*

1. a) Choose the correct alternative. **5**

1) Let $\phi_x(t)$ be a characteristic function of random variable X then $\bar{\phi}_x(t)$ is characteristic function of

a) X^2 b) $1 - X$ c) $-X$ d) none of these

2) Which of the following are Borel sets of real line ?

a) single point sets b) finite sets
c) countable sets d) all the above

3) If A_n is equal to A or B according as n is odd or even, then $\underline{\lim}\, A_n$ =

a) A b) B c) $A \cap B$ d) $A \cup B$

4) Let P(.) is a probability measure defined on $(\Omega, \mathbb{F})$. Then normed property of measure is

a) $P(\Omega) = 0$ b) $P(\Omega) = 1$
c) $P(A) \geq 0$ d) $P(A) = \sum_{i=1}^{n} P(A_i)$

5) If $\{A_n\}$ is monotone increasing sequence of sets then $\lim_{n\to\infty} A_n$ is

a) $\bigcup_{k=1}^{\infty} A_k$ b) $\bigcap_{k=1}^{\infty} A_k$ c) Ω d) ϕ

P.T.O.

b) Fill in the blanks. **5**

1) Power set of finite Ω is the ____________ field.

2) Minimal field containing $A \cap B$ is ______________

3) Characteristic function $\phi_x(0) =$ ________________

4) Let $\{A_n\}$ be a sequence of events such that $\sum_{n=1}^{\infty} P(A_n) < \infty$. Then $P(\overline{\lim} A_n) =$

5) X is said to be integrable if E(X) is _______________

c) State whether the following statements are **true** or **false**. **4**

1) The generalized probability measure has a normed property.

2) Any simple function can be expressed as an elementary function.

3) If $\phi(t)$ is a characteristic function then $|\phi(t)|^2$ is also characteristic function.

4) Every field is a σ – field.

2. a) Answer the following. **6**

i) Define mutual independence and pairwise independence of events.

ii) If X and Y are independent random variables then show that

$E(XY) = E(X)\ E(Y).$

b) Write short notes on the following : **8**

i) Strong law of large numbers.

ii) Kolmogorov's three series criterion for almost sure convergence.

3. a) Let $\{A_n\}$ be a sequence of sets such that lim A_n = A. Show that $\lim A_n^c = A^c$.

b) Find lim inf and lim sup of following sequence of sets

i) $A_n = \left(0,\ 1 + \frac{1}{n}\right)$

ii) $A_n = \left(0,\ 3 + (-1)^n \left(1 + \frac{1}{n}\right)\right)$. **(6+8)**

4. a) Define a field and a σ-field. Prove or disprove: Every field is a σ-field.

b) Establish continuity property of probability measure. **(7+7)**

5. a) Define a random variable. If X is a random variable, examine whether 1-X is also a random variable.

b) Define a measurable function. Examine for a constant function defined on (Ω, $\mathbb{F}$) is measurable. **(7+7)**

6. a) State and prove monotone convergence theorem.

b) Define characteristic function and prove any three properties of characteristic function. **(7+7)**

7. a) Define convergence in probability. State and prove necessary and sufficient condition for convergence in probability.

b) Describe weak law of large numbers (WLLN) for sequence of independent random variables. Prove that WLLN holds for the sequence of Bernoulli random variables. **(7+7)**

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – VI)
Probability Theory (New)

Day and Date : Thursday, 16-4-2015 Total Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt* ***five*** *questions.*
2) Q. No. ***1*** *and Q. No.* ***2*** *are* ***compulsory****.*
3) Attempt ***any three*** *from Q. No.* ***3*** *to Q. No.* ***7****.*
4) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Choose the correct alternative. **5**

1) Let P(.) is a probability measure defined on (Ω, IF). Then P(Ω) = 1 is ______ property of the measure.
   a) Normed b) Non-negativity
   c) Finite additivity d) Sigma additivity

2) If A_n is equal to A or B according as n is odd or even, the $\overline{\lim} A_n$ =
   a) 0 b) 1
   c) $A \cap B$ d) $A \cup B$

3) Let X be a random variable defined on (Ω, IF) then
   a) X^2 is a random variable b) 1 – X is a random variable
   c) |X| is a random variable d) All the above

4) If X_1 and X_2 are independent random variables then $\phi_{x_1+x_2}(t)$ =
   a) $\phi x_1(t) + \phi x_2(t)$ b) $\phi x_1(t)\ \phi x_2(t)$
   c) $\phi x_1(t)$ d) $\phi x_2(t)$

5) Which of the following are Borel sets of real line ?
   a) Single point sets b) Open intervals of type (a, b)
   c) Closed intervals of type [a, b] d) All the above

b) Fill in the blanks : **5**

1) If E(X) is finite then X said to be

2) A finite linear combination of indicators of sets is called ________ function.

3) The minimal σ-field induced by indicator function I_A is

4) The number of points in a set is called

5) A set A is called co-finite set if

c) State whether the following statements are **true** or **false**. **4**

1) The counting measure is a finite measure.

2) Mutual independence implies pairwise independence.

3) If Ω is the set of convergence then $\{X_n\}$ is said to be converge nowhere.

4) Mapping preserves all the set relations.

2. a) Answer the following : **6**

i) For a non-negative random variable X, prove that $E(X) = \int_0^\infty [1 - F(x)]dx$.

ii) Define lim inf and lim sup of sequence of sets $\{A_n\}$.

b) Write short notes on the following : **8**

i) Lebesgue measure.

ii) Indicator function.

3. a) Define monotone decreasing sequence of sets. Prove that if A_n is decreasing sequence of sets then A_n^c is increasing sequence.

b) Find lim inf and lim sup of following sequence of sets.

i) $A_n = \left(1 + \frac{1}{n}, 2 + \frac{1}{n}\right)$

ii) $A_n = \left[a - \frac{1}{n}, a\right]$ **(6+8)**

4. a) Define field. Examine for the class of finite or co-finite sets to be a field.

b) Define probability measure. State and prove monotone property of probability measure. **(7+7)**

5. a) If X and Y are simple random variables then prove that E(X + Y) = E(X) + E(Y).

   b) Let E be an experiment having two outcomes 'success' S and 'failure' F respectively. Let $\Omega = \{S, F\}$ and IF = $\{\phi, S, F, \Omega\}$. Define $X(\omega) = \begin{cases} 1, & \text{if } \omega = S \\ 0, & \text{if } \omega = F \end{cases}$. Examine whether X is random variable with respect to IF. **(7+7)**

6. a) Define almost sure convergence. Prove that almost sure convergence implies convergence in probability.

   b) State Lindberg-Feller form of central limit theorem and deduce the Liapunov's theorem. **(7+7)**

7. a) Define characteristic function of random variable X. Suppose X is Poisson (λ) random variable. Obtain characteristic function of X.

   b) Find the distribution of random variable X when characteristic function is

      i) $\phi_X(t) = \dfrac{1}{1+t^2}$

      ii) $\phi_X(t) = e^{-|t|}$. **(6+8)**

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SLR-BP – 482

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – VI) (Old)
Probability Theory

Day and Date : Thursday, 16-4-2015 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions *: 1) Attempt **five** questions.*
*2) Q. No. **(1)** and Q. No. **(2)** are **compulsory**.*
*3) Attempt **any three** from Q. No. **(3)** to Q. No. **(7)**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) Expectation of random variable $X = X^+ - X^-$ is said to exist if ___________
   a) at least one of $E(X^+)$ or $E(X^-)$ is finite
   b) both $E(X^+)$ and $E(X^-)$ are finite
   c) both $E(X^+)$ and $E(X^-)$ are infinite
   d) none of these

2) Which one of the following statement is correct ?
   a) every field is a σ-field
   b) union of fields is a field
   c) intersection of fields is a field
   d) $\{A, A^C\}$ is a field, where A is proper non-empty subset of Ω .

3) Which one of the following statement is correct ?
   a) $X_n \xrightarrow{a.s.} X \Rightarrow X_n \xrightarrow{P} X$
   b) $X_n \xrightarrow{L} X \Rightarrow X_n \xrightarrow{P} X$
   c) $X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{a.s.} X$
   d) $X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{r} X$

P.T.O.

4) The trivial field is ______________

a) $\{\phi\}$ b) $\{\Omega\}$

c) $\{\phi, \Omega\}$ d) none of these

5) Let X Poisson (λ) variate. Then characteristic function of X is ____________

a) $e^{-\lambda(1-e^{it})}$ b) $e^{\lambda(1-e^{-it})}$

c) $e^{-\lambda(e^{it}-1)}$ d) $e^{\lambda(e^{-it}-1)}$

B) Fill in the blanks : **5**

1) If $\phi_x(t)$ is real then X is ____________

2) A sequence of sets $\{A_n\}$ is said to be monotonic increasing if ____________

3) If $X_n \xrightarrow{r} X$ then $E|X_n|^r \to$ ____________

4) If A is finite set with n elements then power set of A contains __________ elements.

5) If P $(\cdot)$ is a probability measure then $P(A^C) =$ ____________

C) State whether the following statements are **TRUE** or **FALSE**. **4**

1) Almost sure convergence always implies convergence in probability.

2) Every field is a σ-field.

3) Mutual independence implies Pairwise independence.

4) Every field contains empty set ϕ.

2. a) Answer the following : **6**

i) Define field and σ-field.

ii) Prove that every σ-field is a field.

b) Write short notes on the following : **8**

i) Mutual and Pairwise independence.

ii) Strong law of large numbers.

3. a) If F_1 and F_2 are fields. Show that

   i) $F_1 \cap F_2$ is a field.

   ii) $F_1 \cup F_2$ is not a field.

   b) Find lim A_n if exist. $A_n = \left(1+\frac{1}{n}, 2+\frac{1}{n}\right)$ **(8+6)**

4. a) State and prove continuity property of probability measure.

   b) If $X_n \leq Y$ and Y is integrable then show that $E(\overline{\lim} X_n) \geq \overline{\lim} E(X_n)$. **(7+7)**

5. a) Prove that $X_n \xrightarrow{P} 0$ if and only if $E\left(\frac{|X_n|}{1+|X_n|}\right) \to 0$ as $n \to \infty$.

   b) Let $\{X_n\}$ be a sequence of random variables such that $X_n \xrightarrow{L} X$ and c be a constant. Show that

   i) $X_n + c \xrightarrow{L} X + c$

   ii) $c X_n \xrightarrow{L} cX, c \neq 0$. **(6+8)**

6. a) State Kolmogrov's three series criterion for almost sure convergence.

   b) Let $\{A_n\}$ be a sequence of events such that $\sum_{n=1}^{\infty} P(A_n) < \infty$. Show that

   $P(\overline{\lim} A_n) = 0$. **(6+8)**

7. a) Define characteristic function. Suppose X is B(n, p) random variable. Obtain characteristic function of X.

   b) State inversion formula and obtain the probability distribution of random variable corresponding to characteristic function $\phi_X(t) = \frac{1}{1+t^2}$. **(6+8)**

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SLR-MB – 613

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2016
STATISTICS (Paper – VI)
Probability Theory (New CBCS)

Day and Date : Wednesday, 30-3-2016 Total Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions *: 1) Attempt* ***five*** *questions.*

2) Q. No. ***(1)*** *and Q. No.* ***(2)*** *are* ***compulsory.***

3) Attempt ***any three*** *from Q. No.* ***(3)*** *to Q. No.* ***(7)****.*

4) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Choose the correct alternative : **5**

1) If $\mathbb{F}_1 = \{A, A^C, \phi, \Omega\}$ and $\mathbb{F}_2 = \{B, B^C, \phi, \Omega\}$. Then ___________ .

a) Only $\mathbb{F}_1$ is a field b) Only $\mathbb{F}_2$ is a field

c) $\mathbb{F}_1 \cup \mathbb{F}_2$ is a field d) $\mathbb{F}_1 \cap \mathbb{F}_2$ is a field

2) A power set of finite Ω is ___________ field.

a) Smallest b) Largest

c) Degenerate d) None of these

3) If X is random variable then ___________

a) | X | is also random variable b) X^+ and X^- are random variables

c) 1 – X is random variable d) All the above

4) Let $\{X_n\}$ be a sequence of iid random variables with $Var(X_n) < \infty$ then CLT ____

a) Holds if $E(X_n) = 0$ for all n b) Holds if $E(X_n) \to 0$ as $n \to \infty$

c) Does not hold d) None of these

5) Expectation of X ___________

a) Always exist b) May not exist

c) Is always zero d) None of these

b) Fill in the blanks : **5**

1) The minimal σ-field containing $A \cap B$ is ____________

2) Characteristic function of Binomial random variable is ______________

3) Borel sets are subsets of ___________

4) Sigma additivity property of probability measure P(.) is given as ____________

5) WLLN states that sample mean converges in ____________ to population mean.

c) State whether the following statements are **True** or **False** : **4**

1) Counting measure is a finite measure.

2) If ϕ (t) is a characteristic measure then $|\phi(t)|^2$ is also characteristic function.

3) Mapping preserves the set relations.

4) If $X_n \xrightarrow{P} X$ then $X_n \xrightarrow{L} X$.

2. a) Define : **6**

i) Field ii) σ – field

Give an example of a field which is not a σ – field.

b) Write short notes on the following : **8**

i) Generalized probability measure

ii) Central Limit Theorem (CLT)

3. a) Show that intersection of two fields is a field. Give an example to show that union of two fields may not be a field.

b) Define probability measure. Prove that $P(\lim A_n) = \lim P(A_n)$. **(7+7)**

4. a) If X and Y are two random variables then prove that Max (X, Y) and Min (X, Y) are also random variables.

   b) If X and Y are two independent random variables then prove that E(XY) = E(X)E(Y). **(7+7)**

5. a) Define various modes of convergence of sequence of random variables.

   b) Prove that $X_n \xrightarrow{P} 0$ if and only if $E\left[\frac{|X_n|}{1+|X_n|}\right] \to 0$ as $n \to \infty$. **(6+8)**

6. a) State and prove monotone convergence theorem.

   b) Prove that X is integrable if and only if | X |, is integrable. **(7+7)**

7. a) Describe weak law of large numbers for a sequence of random variables. Prove that WLLN holds for the sequence of Bernoulli random variables.

   b) State inversion formula and obtain distribution corresponding to characteristic function $\phi(t) = \exp(-|t|)$. **(7+7)**

SLR-BP – 477

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – VI)
Probability Theory (New)

Day and Date : Thursday, 16-4-2015 Total Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. a) Choose the correct alternative. **5**

1) Let P(.) is a probability measure defined on (Ω , IF). Then P(Ω) = 1 is ______ property of the measure.
a) Normed b) Non-negativity
c) Finite additivity d) Sigma additivity

2) If A_n is equal to A or B according as n is odd or even, the $\overline{\lim} A_n =$
a) 0 b) 1
c) $A \cap B$ d) $A \cup B$

3) Let X be a random variable defined on (Ω , IF) then
a) X^2 is a random variable b) 1 – X is a random variable
c) |X| is a random variable d) All the above

4) If X_1 and X_2 are independent random variables then $\phi_{x_1+x_2}(t)$ =
a) $\phi x_1(t) + \phi x_2(t)$ b) $\phi x_1(t)\ \phi x_2(t)$
c) $\phi x_1(t)$ d) $\phi x_2(t)$

5) Which of the following are Borel sets of real line ?
a) Single point sets b) Open intervals of type (a, b)
c) Closed intervals of type [a, b] d) All the above

P.T.O.

b) Fill in the blanks : **5**

1) If E(X) is finite then X said to be

2) A finite linear combination of indicators of sets is called ________ function.

3) The minimal σ-field induced by indicator function I_A is

4) The number of points in a set is called

5) A set A is called co-finite set if

c) State whether the following statements are **true** or **false**. **4**

1) The counting measure is a finite measure.

2) Mutual independence implies pairwise independence.

3) If Ω is the set of convergence then $\{X_n\}$ is said to be converge nowhere.

4) Mapping preserves all the set relations.

2. a) Answer the following : **6**

i) For a non-negative random variable X, prove that $E(X) = \int_0^\infty [1 - F(x)]dx$.

ii) Define lim inf and lim sup of sequence of sets $\{A_n\}$.

b) Write short notes on the following : **8**

i) Lebesgue measure.

ii) Indicator function.

3. a) Define monotone decreasing sequence of sets. Prove that if A_n is decreasing sequence of sets then A_n^c is increasing sequence.

b) Find lim inf and lim sup of following sequence of sets.

i) $A_n = \left(1 + \frac{1}{n}, 2 + \frac{1}{n}\right)$

ii) $A_n = \left[a - \frac{1}{n}, a\right]$ **(6+8)**

4. a) Define field. Examine for the class of finite or co-finite sets to be a field.

b) Define probability measure. State and prove monotone property of probability measure. **(7+7)**

5. a) If X and Y are simple random variables then prove that $E(X + Y) = E(X) + E(Y)$.

   b) Let E be an experiment having two outcomes 'success' S and 'failure' F respectively. Let $\Omega = \{S, F\}$ and $IF = \{\phi, S, F, \Omega\}$. Define $X(\omega) = \begin{cases} 1, & \text{if } \omega = S \\ 0, & \text{if } \omega = F \end{cases}$. Examine whether X is random variable with respect to IF. **(7+7)**

6. a) Define almost sure convergence. Prove that almost sure convergence implies convergence in probability.

   b) State Lindberg-Feller form of central limit theorem and deduce the Liapunov's theorem. **(7+7)**

7. a) Define characteristic function of random variable X. Suppose X is Poisson (λ) random variable. Obtain characteristic function of X.

   b) Find the distribution of random variable X when characteristic function is

      i) $\phi_X(t) = \dfrac{1}{1 + t^2}$

      ii) $\phi_X(t) = e^{-|t|}$. **(6+8)**

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Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2014
STATISTICS (Paper – VI)
Probability Theory

Day and Date : Tuesday, 22-4-2014 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select correct alternative : **5**

1) If X is r.v. then ____________
a) X^2 is r.v. b) aX + b is r.v., a, b constants
c) | X | is r.v. d) all the above

2) If $\{A_n\}$ is ↓ sequence of sets then $\overline{\lim}\ A_n$ is____________
a) Ω b) $\bigcap_{n=1}^{\infty} A_n$ c) $\bigcup_{n=1}^{\infty} A_n$ d) none of these

3) Characteristic function of standard exponential distribution is ____________
a) (1 – it) b) $(1 - it)^2$ c) $\frac{1}{(1-it)}$ d) $\frac{1}{(1-it)^2}$

4) The range of indicator function I_A is ____________
a) {–1, 1} b) {0, 1} c) {0, ∞} d) {– ∞, ∞}

5) An elementary function is a ____________ linear combination of indicator functions.
a) countable b) finite
c) both a) and b) d) neither a) nor b)

B) Fill in the blanks : **5**

1) $|\phi_X(t)|$ is bounded by __________

2) Let $\{A_n\}$ be a sequence of monotonic increasing sets then $\lim\limits_{n\to\infty} A_n$ = __________

3) The smallest field containing ϕ and Ω is __________

4) Monotone convergence theorem states that __________

5) Convergence in law __________ convergence in mean.

C) State whether the following statements are **true** or **false** : **4**

1) The product of any finite number of characteristic functions is also a characteristic function.

2) $|X|$ is integrable does not imply X is integrable.

3) A probability measure is non-decreasing function.

4) A minimal field need not be unique.

2. a) Answer the following : **6**

i) Show that X is integrable iff $|X|$ is integrable.

ii) If $X_n \xrightarrow{P} X$ and $X_n \xrightarrow{P} X'$ then show that X and X' are equivalent.

b) Write short notes on the following : **8**

i) Counting measure

ii) Mixture of probability measures.

3. a) Give two definitions of field and establish their equivalence.

b) Give an example of a field which is not a σ-field.

c) Prove or disprove : Union of two fields is a field. **(6+4+4)**

4. a) Define limit of sequence of sets. Prove or disprove : If $\lim A_n$ exists then $\lim A_n^c$ also exists.

b) Find $\underline{\lim}\, A_n$ of the following :

i) $A_n = \left(1+\hbar,\ 3+\frac{2}{n}\right), n \geq 1.$

ii) $A_n = \left(0, 1-\frac{1}{n}\right), n \geq 1.$ **(8+6)**

5. a) Define expectation of
   i) simple r.v.
   ii) non-negative r.v.
   iii) arbitrary r.v.

   b) State and prove Fatou's lemma. **(6+8)**

6. a) Define :
   i) Weak Law of Large Numbers (WLLN)
   ii) Strong Law of Large Numbers (SLLN).

   b) Define characteristic function of r.v.X. Show that characteristic function $\phi_X(t)$ is real if and only if X is symmetric about origin.

   c) Let X be B (n, p) random variable. Obtain $\phi_X(t)$. **(6+4+4)**

7. a) Define :
   i) Convergence in probability
   ii) Almost sure convergence.

   Prove that $X_n \xrightarrow{a.s.} X$ implies $X_n \xrightarrow{P} X$.

   b) If $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$ then show that

   i) $X_n + Y_n \xrightarrow{P} X + Y$

   ii) $X_n \cdot Y_n \xrightarrow{P} XY$ **(6+8)**

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