Shivaji University, Kolhapur

Question Bank For Mar 2022 (Summer) Examination

M.Sc.(Statistics/Applied Statistics and Informatics) Exam

Subject Code : 74910/83441/74977

Subject Name : Estimation Theory

Short answer questions (2 marks)

1 Define sufficient statistic. Obtain a sufficient statistic for the variance of a normal distribution based on a sample of size n when mean is known.

2 Define complete family of distributions.

3 State Basu's theorem.

4 Define minimal sufficient statistic. Give an example.

5 Define power series family of distributions.

6 Define curved exponential family. Give an example.

7 Define ancillary statistics. State an ancillary statistic for μ based on two observations X_1 and X_2 from $N(\mu, 1)$ distribution.

8 Define Pitman family of distributions. Give an example.

9 Explain the concept of sufficiency with an example.

10 Define complete sufficient statistic.

11 Give two examples of families of distributions which are not exponential families.

12 Give two examples of two-parameter family of distributions that is not exponential families.

13 Define bounded completeness. Give an example of a bounded compete family.

14 State Rao-Blackwell theorem.

15 Define minimum variance unbiased estimator.

16 Let $X \sim B(n, p)$, obtain UMVUE of p.

17 Illustrate with an example that unbiased estimators need not be unique.

18 Define Fisher information. Obtain Fisher information contained in a single observation drawn from $B(n, p)$.

19 If X follows $P(\lambda)$, obtain Fisher information of λ^2.

20 Explain Chapman-Robbins-kiefer bound.

21 If X ~ NB(r, p), show that $(r-1)/(X+r-1)$ is an unbiased estimator of p.

22 State one application of Chapman-Robinson bounds.

23 What is the Fisher's information contained in a single observation drawn from $N(\mu, 1)$ distribution?

24 State the invariance property of MLE.

25 Let $X \sim B(1, p)$, where $p \in (\frac{1}{4}, \frac{3}{4})$. Obtain MLE of p based on X.

26 Define kernel and U-statistic for an estimable parameter $g(F)$.

27 Explain the method of obtaining moment estimators.

28 Define one-sample U-statistic. Give an example.

29 Let X_1, X_2 be a random sample from pdf $f_\theta(x) = \frac{\theta}{x^2}, x > \theta, \theta > 0; = 0$, otherwise.What is the MLE of θ?

30 Which statistic is used in minimum chi-square method to estimate a parameter?
31 What is likelihood function?
32 Is MLE a function of minimal sufficient statistic? Justify your answer.
33 Define Bayes rule and Bayes risk.
34 Define prior and posterior distributions.
35 Define improper prior. Give an example.
36 Define conjugate family of distributions. Give an example.
37 Define (i) a loss function (ii) absolute error loss function.
38 State Bayes estimators under squared error loss and absolute error loss functions.

Long answer questions (8 marks)

1 Let X_1, X_2, X_3 be a random sample obtained from $B(1,\theta)$. Show that $X_1 + 2X_2$ is not sufficient statistic for θ.
2 State and prove Neyman's factorization theorem for discrete family of distributions.
3 Define one parameter exponential family of distributions. Obtain minimal sufficient statistic for this family.
4 Let $X_1, X_2, \cdots, X_n$ be a random sample from PMF $P_N(x) = \frac{1}{N}, x = 1, 2, \cdots N$; and $= 0$ otherwise. Obtain a complete sufficient statistics for the family of PMFs $\{P_N(x), N \geq 1\}$.
5 Define ancillary statistic. State and prove Basu's theorem.
6 Define minimal sufficient statistic. Obtain the same for θ, when a sample of size n is drawn from $U(-\theta, \theta)$.
7 Suppose a sample of size n is drawn from $P(\lambda)$. Obtain complete sufficient statistic of λ.
8 Let $X_1, X_2, \cdots, X_n$ be independent random variables each having U(0, θ) distribution. Verify whether T_1= min $(X_1, X_2, \cdots, X_n)$ is sufficient for θ.
9 Define (i) Completeness (ii) Bounded completeness
Suppose $X_1, X_2, \cdots, X_n$ is a random sample from $N(\theta, \theta^2)$. Show that $T = (\sum X_i, \sum X_i^2)$ is sufficient but not complete.
10 X_1 and X_2 have Bernoulli distribution with parameter p. Verify whether (i) $X_1 + X_2$ (ii) $X_1 + 2X_2$ are minimal sufficient statistics.
11 Define sufficient statistic. Obtain sufficient statistics for the distributions with following probability density functions.
(i) $f_{\alpha,\beta}(x) = \beta e^{-\beta(x-\alpha)}$, $x > \alpha$
(ii) $f_{\alpha,\beta}(x) = \frac{1}{\beta-\alpha}$, $\alpha < x < \beta$
12 Explain bounded completeness through an example.
13 Discuss Pitman family of distributions in detail.
14 Describe curved exponential family.
15 Let $X_1, X_2, \cdots, X_n$ be a random sample from pdf
$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(\log x - \mu)^2}$; $x > 0$. Obtain sufficient statistic for (i) μ when σ^2 is known, (ii) σ^2 when μ is known.

16 Define ancillary statistic. Suppose X_1 and X_2 are iid observations from the pdf $f_\alpha(x) = \alpha x^{\alpha-1} e^{-x^\alpha}, x > 0, \alpha > 0; = 0$, otherwise. Show that $\log X_1 / \log X_2$ is an ancillary statistic.

17 Describe Fisher Information Function.

18 Let $X_1, X_2, \cdots, X_n$ be a random sample from $N(\mu, \sigma^2)$. Obtain Fisher information matrix $I_x(\mu, \sigma^2)$.

19 State and prove Lehman-Scheffe theorem.

20 Define UMVUE. State and prove Rao-Blackwell theorem.

21 State and prove a necessary and sufficient condition for an estimator of a parametric function $\psi(\theta)$ to be UMVUE.

22 Obtain UMVUE of $P(X = 1)$ based on a random sample of size n, where X has $P(\lambda)$ distribution.

23 X and Y are independent random variables such that $P(X = 1) = 2\theta$ and $P(Y = 1) = \theta, 0 < \theta < 1/2$. Show that the linear estimator $U(X, Y)$ is unbiased for θ iff $U(X, Y) = aX + (1 - 2a)Y$ for some real number a.

24 Given a random sample from $N(0, \theta)$ distribution, $0 < \theta < \infty$. Obtain UMVUE for $\sqrt{\theta}$.

25 State and prove Chapman-Robbins-Kiefer inequality.

26 Let $X_1, X_2, \cdots, X_n$ be independent Bernoulli random variables with probability of success p. Let $g(p) = \frac{1}{p}$, Show that there is no UMVUE of $g(p)$.

27 If $X_1, X_2, \cdots, X_n$ is a random sample from $U(\theta_1, \theta_2)$, $\theta_1 \leq x \leq \theta_2$. Obtain UMVUE for θ_2.

28 Let $X_1, X_2, \cdots, X_n$ are iid $P(\theta)$ random variables. Show that the variance of T_n (an unbiased estimator of θ) $T_n = (1 - \frac{1}{n})^{\sum_{i=1}^{n} x_i}$ does not attain the Bhattacharya bound of order 2.

29 Define unbiased estimator and state its properties.

30 Define UMVUE. Suppose $X_1, X_2, \cdots, X_n$ are iid random variable from Poisson(θ). Obtain UMVUE of $e^{-\theta}$.

31 State Cramer-Rao lower bound (CRLB). Obtain Bhattacharya lower bounds and show that it reduces to CRLB for k=1.

32 Check whether the family of distributions $\{f_\theta, \theta > 0\}$, where $f_\theta(x) = \theta^{-1} e^{-x/\theta}, x > 0$ and = 0 otherwise, satisfies the regularity conditions of CR inequality. If so, obtain the CR lower bound for the variance of an unbiased estimator of θ.

33 State and Prove Crammer-Rao inequality stating the regularity conditions.

34 Let X~ Geometric(p) distribution. Obtain UMVUE of p based on a random sample of size n.

35 Explain method of moment estimators. Obtain moment estimators of a and b of U(a, b) distribution based on random sample of size n.

36 Define kernel and U-statistic. Obtain U-statistics based on a sample of size *n* (>1) for mean and variance of a distribution for which the second moment exists.

37 Describe method of scoring to obtain MLE with suitable example.

38 Let $X_1, X_2, \cdots, X_n$ be a random sample from N (θ, θ) distribution, $0 < \theta < \infty$. Obtain MLE of θ.

39 Define MLE. State and prove invariance property of MLE.

40 Let $X_1, X_2, \cdots, X_n$ be a random sample from $N(\mu, \sigma^2)$ distribution. Obtain MLEs of μ and σ^2.

41 Explain the method of maximum likelihood for estimating the parameter of a distribution.

42 Suppose $X_1, X_2, \cdots, X_n$ is a random sample from $f_\theta(x) = e^{-(x-\theta)}, x \geq \theta$. Obtain MLE of θ. Is it unbiased? Justify.

43 Define MLE. Let $X_1 \sim N(\theta, 1), X_2 \sim N(2\theta, 1), -\infty < X < \infty$. Find the MLE for θ.

44 If $\pi_1(\theta) = \pi_2(\theta) = \theta, \pi_3(\theta) = 1 - 2\theta$, based on n_1, n_2 and n_3 frequencies. Obtain minimum chi- square and modified minimum chi- square estimator of θ.

45 Obtain the expression for the variance of U-statistic.

46 Obtain MLE of θ based on a sample of size n from a population with pdf $f(x) = \theta x^{\theta - 1}, 0 < x < 1, \theta > 0$. Is it unbiased for θ? Justify your answer.

47 Describe method of moments. Let $X_1, X_2, \ldots, X_n$ be a sample from G(α, *β)* distribution. Obtain the method of moment estimator of (α, *β).*

48 A random sample of size n is drawn from G(α, β) distribution. Find MLE of α when β is known.

49 Let $X_1, X_2, \cdots, X_n$ be a random sample from $N(\theta, 1), \theta \in R$, distribution. The prior distribution of θ is $N(0,1)$. Find the Bayes estimator of θ under squared error loss function.

50 Show that the Bayes estimator under absolute error loss function is the median of the posterior distribution.

51 Define conjugate family of priors. Show that $G(\alpha, \beta)$ belongs to conjugate family if X is a random variable from $P(\theta)$.

52 Describe squared error loss functions and absolute error loss functions. Obtain Bayes estimator under squared error loss.

53 Let a random sample of n independent observations are available from $P(\theta)$. Using prior distribution of θ as $G(\propto, p)$, obtain Bayes estimator of θ.

54 Explain with examples : (i) non informative prior and (ii) Jeffrey's prior.

55 Describe various types of priors.

Short notes (4 Marks each)

1 Complete sufficient statistic

2 Pitman family of distributions

3 Bounded Completeness

4 Minimal sufficient partition

5 Sufficient statistic and Neyman factorization theorem

6 Crammer-Rao inequality

7 Fisher information function

8 Bhattacharya bounds

9 Chapman-Robbins-Kiefer bound

10 Unbiased estimator

11 Minimum chi-square estimation

12 Method of scoring

13 Method of moment estimation

14 Properties of an MLE

15 U-statistics

16 Conjugate prior and non informative prior
17 Jeffery’s least favorable prior
18 Procedure to obtain Bayes estimators
19 Bayes estimation under absolute error loss function
20 Posterior distributions
21 Bayes and minimax rules

*****