Shivaji University Kolhapur

B.Sc. Part III Physics CSBC ()Sem V

Physics Paper- X: DSC-E2 Quantum mechanics

Question Bank

Unit I :Matter waves

Select correct alternatives for the following (The correct alternatives in red color)

(i)....... waves are packed over a small region called wave packet.

(a) standing waves (b) progressive waves

(c) matter waves (d) interfering waves

(ii) The particle parameters (E, p) are related with wave parameters(ω,k) by the relations.........

(a) E=ħk and p = ħw (b) E = ħw and p =ħk

(c) E=ħ/k and p=ħ/w (d) E= ħ/w and p=ħ/k

(iii) ---------exhibit dual nature viz. wave and particle nature.

(a) Both photons and material particles (b) photons

(c) material particles (d) waves

(iv) de Broglie wavelength (λ) is given by----------

(a) λ=h/ P (b) λ = P/ h

(c) λ=hp (d)λ = C/υ

(v) Phase velocity (u) is given by---------

(a) u= $\Delta\omega/\Delta k$ (b) u = $\Delta k/\Delta\omega$

(c)u= ω/k (d) u= k/ω

(vi) Group velocity (Vg) is given by----------

(a)$V_g=\Delta\omega / \Delta k$ (b) Vg= $\Delta k/ \Delta\omega$

(c) $V_g =\omega/k$ (d) Vg = k /w

(vii) Relation between group velocity (V_g) and particle velocity (V) is--------------

(a) Vg>V (b) Vg<V (c) Vg = V (d) Vg ≠V

(viii) Uncertainty principle is expressed as,

(a) $\Delta E\cdot\Delta p\geq \hbar$ (b) $\Delta E\cdot\Delta x \geq \hbar$

(c)$\Delta x\cdot\Delta t\geq\hbar$ (d) $\Delta x\cdot\Delta p \geq \hbar$

- ***Short answer questions (5 marks)***

1. Discuss about wave particle duality.
2. State and explain de Broglie hypothesis of matter waves

3. Derive an expression for matter waves.
4. Explain the concept of wave packet.
5. State and explain uncertainty principle.
6. Calculate wavelength of electron travelling with a speed of 2.65 x 10^6 m/s.

7.What is the speed of electron having λ=250 nm

- ***Long answer questions(10 marks)***

1. Define and obtain expressions for phase velocity and group velocity.
2. Obtain relation between particle velocity and the group velocity.
3. Explain in details, how Davisson and Germer experiment proves deBroglie hypothesis.
4. On the basis of uncertainty principle, show that the electrons can't exist inside the nucleus.
5.What is the velocity of electron having de Broglie wave length approximately equal to chemical bond length of 1.2 x 10^{-10}m.
6.An electron is passing through a circular slit of radius 1 cm. Calculate the uncertainty in the momentum of electron.
7. Calculate de Broglie wavelength of an electron accelerated through a potential of 55 volts.

Unit II : Schrodinger wave equation

Select correct alternatives for the following (The correct alternatives in red color)

(i) The wave function ψ, obeys the boundary condition.--------

(a) $|\psi| \rightarrow 0$ as $r \rightarrow \infty$ (b) $|\psi| \rightarrow \infty$ as $r \rightarrow 0$

(c) $|\psi| \rightarrow 0$ as $r \rightarrow 0$ (d) $|\psi| \rightarrow \infty$ as $r \rightarrow \infty$

(ii) The normalisation condition for the wave function is

(a) $\int_{-\infty}^{+\infty} \psi^* \psi d^3r = 0$ (b) $\int_{-\infty}^{+\infty} \psi^* \psi d^3r = 1$

(c) $\int_{-\infty}^{+\infty} \psi^* \psi d^3r \leq 1$ (a) $\int_{-\infty}^{+\infty} \psi^* \psi d^3r \geq 1$

(iii) The orthogonal condition for the wave functions ψ_1 (x) and ψ_2 (x) is-----

(a) $\int_a^b \psi_1^*(x) \cdot \psi_2^*(x) dx = 0$ (b) $\int_a^b \psi_1(x) \cdot \psi_2(x) dx = 0$

(c) $\int_a^b \psi_1^*(x) \cdot \psi_2(x) dx = 0$ (d) $\int_a^b \psi_2(x) \cdot \psi_2^*(x) dx = 1$

(iv) The expectation value of a function f (x), with normalised wave function (x) of a system is...

(a) $\langle f(x)\rangle = \int_{-\infty}^{+\infty} \psi^*(x) f(x)\, \psi(x)\, dx$ (b) $\langle f(x)\rangle = \int_{-\infty}^{+\infty} \psi(x) f(x)\, \psi(x)\, dx$

(c) $\langle f(x)\rangle = \int_{-\infty}^{+\infty} \psi^*(x) f(x)\, \psi^*(x)\, dx$ (d) $\langle f(x)\rangle = \int_{-\infty}^{+\infty} \psi(x) f(x)\, \psi^*(x)\, dx$

(v) In terms of the momentum (p) and propagation vector (k), the relation for de Broglie wave is..........

(a) $p=\hbar k$ (b) $p=\hbar/k$ (c) $p=\hbar\omega$ (d) $p= k/\hbar$

(vi) Einstein's frequency relation is.........

(a) $E=h\omega$ (b) $E=\hbar\omega$ (c) $E=\hbar\upsilon$ (d) $E=h/\upsilon$

(vii) The probability current density is given by..........

(a) $J= 2m/i\hbar\, [\psi^*\nabla\psi - \psi\nabla\psi^*]$ (b) $J= i\hbar/2m\, [\psi\nabla\psi^* - \psi^* \nabla\psi]$

(c) $J= i\hbar/2m\, [\psi^*\nabla\psi - \psi\nabla\psi^*]$ (d) $J= 2m/i\hbar\, [\psi\nabla\psi^* - \psi^* \nabla\psi]$

- ***Short answer questions (5 marks)***

1. Give physical interpretation of the wave function and state the conditions that the wave function should satisfy.
2. Derive Schroedinger's time dependent wave equations for the matter wave in 1-D
3. Derive Schroedinger's time independent wave equations for the matter wave in 1-D
4. Write note on:
   (a) Expectation value of the dynamical variables.
   (b) Orthogonal and normalisation conditions of the wave functions.
5. If (P_x) is the x-component of the momentum and V is the potential at the position x, then prove that,
   (a) $d/dt\langle x\rangle = \langle P_x\rangle/m$ where, m is mass of the particle
   (b) $d/dt\langle P_x\rangle=-\langle dV/ dx\rangle$

- ***Long answer questions(10 marks)***

1. Derive Schroedinger's time dependent wave equations for the matter wave in 3-D.
2. Derive Schroedinger's time independent wave equations for the matter wave in 3-D.
3. From Schroedinger's time dependent wave equation derive an expression for J
4. Show that the probability density $P = \psi^* \psi$ and probability current density $J= i\hbar/2m\, [\psi\nabla\psi^* - \psi^* \nabla\psi]$, satisfy the equation of continuity and thereby explain the physical significance of equation of continuity

Unit -III : Operators

Select correct alternatives for the following (The correct alternatives in red color)

(i) f(x) is an eigen function corresponding to an operator Ā, then Ā f(x) is----

(a) linearly related with f (x) (b) non-linearly related with f (x)

(c) f (x) (d) d/ dx f(x)

(ii) Linear momentum operator p_x is given by-------.

(a) $p_x = i\hbar (d/dx)$ (b) $p_x = - i\hbar (d/dx)$

(c) $p_x = i\hbar (d^2/dx^2)$ (d) $p_x = - i\hbar (d^2/dx^2)$

(iii) The Hamiltonian operator H is given by

(a) $H = -i\hbar\nabla$ (b) $H = i\hbar (d/dt)$

(c) $H = - \hbar^2/2m \nabla^2 + V(r)$ (d) $H = \hbar^2/2m \nabla^2 - V(r)$

(iv) The kinetic energy operator is given by.......

(a) $T = i\hbar (d/dt)$ (b) $T = - \hbar^2/2m \nabla^2 + V(r)$

(c) $T = \hbar^2/2m \nabla^2 + V(r)$ (d) $H = - \hbar^2/2m \nabla^2$

(v) The energy operator E is given by----------

(a) $\bar{E} = -i\hbar\nabla$ (b) $\bar{E} = i\hbar\nabla$

(c) $\bar{E} = i\hbar (d/dt)$ (d) $\bar{E} = -i\hbar (d/dt)$

(vi) Antisymmetric function of wave function ψ (x) is……

(a) - ψ (x) (b) ψ (x) (c)- ψ (-x) (d) ψ (-x)

(vii) z-component of angular momentum operator is……………

(a) $L_z = m\hbar$ (b) $L_z = -m\hbar$ (c) $L_z = -i\hbar (d/d\Phi)$ (d) $L_z = ih (d/d\Phi)$

(viii) The eigen value of operator L^2 is given by………

(a) $< L^2 > = m\hbar$ (b) $<L^{2>} = -m\hbar$

(c) $< L^2 > = l(l+1)\hbar^2$ (d) $< L^2 > = - l(l+1)\hbar^2$

(ix) Commutation relations among position and momentum operator are expressed as.........

(a) $[x_i, p_j] = i\hbar\, \delta_{ij}$ (b) $[x_i, p_j] = -i\hbar\, \delta_{ij}$

(c) $[x_i, p_j] = i\hbar$ (d) $[x_i, p_j] = 0$

(x) Raising operator in quantum mechanics is.........

(a) $L_+ = L_X + iL_Y$ (b) $L_+ = L_x - iLy$ (c) $L_+ = L_x + iL_z$ (d) $L_+ = L_x - iL_z$

- ***Short answer questions (5 marks)***

1.Define an operator. Hence, obtain expressions for

(i) linear momentum operator, (ii) total energy operator, (iii) parity operator.

2.Derive commutation relations for L_x, L_y and L_z
3. Define ladder operators and explain their effects.
4. Write a note on commutation rules for operators

- ***Long answer questions(10 marks)***

1.Derive expressions for the operators L_x, L_y, L_z and L^2 in spherical polar co-ordinates
2. Find eigen values of L_z and L^2.
3. Show that, if operators A and B commute with an operator C, then commutator of A and B also commutes with C.
4. Obtain expressions for operators L_x, L_y and L_z in cartesian co-ordinates

Unit IV: Application of schrodinger equation

Select correct alternatives for the following (The correct alternatives in red color)

(i)The energy spectrum of a particle in one-dimensional rigid box has the nature of--------

(a) infinite sequence of discrete energy levels
(b) infinite sequence of equidistant energy levels
(c) exponentially increasing
(d) exponentially decreasing

(ii) The non-degenerate state of the energy possessed by a particle in three-dimensional rigid box is given by.....

(a) $n_x= 3, n_y= 3, n_z = 3$ (b) $n_x = 2, n_y = 2, n_z = 2$
(c) $n_x= 4, n_y= 4, n_z= 4$ (d) $n_x = 5, n_y = 5. n_z = 5$

(iii) The parity of the wave function is even when.........

(a) $\psi(-x)= -\psi(x)$ (b) $\psi(x)= x^3$ (c)$\psi(-x)= \psi(x)$ (d) $\psi(x)= x$

(iv) The energy levels possessed by a linear harmonic oscillator are......

(a) infinite sequence of discrete energy levels
(b) exponential in nature
(c) infinite sequence of discrete equidistant energy levels
(d) continuous energy levels

(v) The zero point energy of linear harmonic oscillator is

(a) $E_0=0$ (b) $E_0 = \hbar\omega$ (c) $E_o = 2 \hbar\omega$ (d) $E_o= 1/2 \hbar\omega$

(vi) A standing wave is formed between two supports at x = 0 and x = L with one loop, then energy possessed by a vibrating the particle of mass (m) which produces this standing wave is.-----

(a) $E = h^2 \pi^2 / 2mL^2$ (b) $E = \hbar^2 \pi^2 / 2mL^2$

(c) $E = 1/2\ \hbar\omega$ (d) $E = \hbar\omega$

(vii) Which of the following set will be obeyed by the magnetic orbital quantum number, when $l = 2$?

(a) $m_l = 1, 0, -1$ (b) $m_l = 2, 1, 0, -1, -2$

(c) $m_l = 2, 0, -2$ (d) $m_l = 0, 1, 2$

(viii) In a normal state of the atom, the number of electrons in a sub-shell of the atom is given by

(a) $l\sqrt{l+1}$ (b) $(2l+1)$ (c) $2(2l+1)$ (d) $l+1/2$

- ***Short answer questions (5 marks)***

1.Draw energy level diagram for linear harmonic oscillator.
2. Draw energy level diagram for hydrogen atom.
3.Explain the tunnel effect in case of rectangular potential barrier
4. Derive and explain zero point energy of an harmonic oscillator
5. Write notes on:
   (i) Zero point energy,
   (ii) Tunnel effect,
   (iii) Reflection and transmission coefficients in rectangular potential barrier,
   (iv) Degenerate states of the energy levels of the particle in three-dimensional rigid box

- ***Long answer questions(10 marks)***

1.Calculate the reflection and transmission coefficients of electron through one-dimensional rectangular potential barrier
2. What is reduced mass of electron in Hydrogen atom and why this term is used?
3. Solve radial part of the Schroedinger equation for hydrogen atom. neglecting electron spin angular momentum and obtain the energy eigen values. Explain the degeneracy in the spectrum.
4. Write down Hamiltonian for hydrogen atom and hence set-up Schroedinger wave equation in spherical polar co-ordinates.