SF – 530

Total No. of Pages : 3

Seat No.	01163

M.Sc. (Part - I) (Semester - I) (CBCS) (New) Examination, November - 2019

STATISTICS/APPLIED STATISTICS AND INFORMATICS (Paper - III)

Distribution Theory

Sub. Code : 74909/74976

Day and Date : Wednesday, 27 - 11 - 2019 **Total Marks : 80**

Time : 11.00 a.m. to 02.00 p.m.

Instructions :
1) **Question No. 1 is compulsory.**
2) **Attempt any four qustions from question numbers 2 to 7.**
3) **Figures to the right indicate full marks.**

Q1) Answer the following : ***[8 × 2 = 16]***

a) Define mixture of distributions.

b) Give an algorithm for generating random numbers from mixture of two distributions.

c) Let F(x) be a *cdf*. Verify whether $(F(x))^{\alpha}, \alpha > 0$ is a *cdf* or not.

d) Define mixed moments. How are they computed?

e) Define conditional expectation and conditional variance.

f) Establish :

$E(X) = E_Y E_X(X14)$.

g) Let X~ Exp(θ_1) and Y~ Exp(θ_2) and are independent. Obtain pdf of M = Max (X, Y).

h) Define scale parameter. Give an example.

Q2) a) Let X be a random variable having symmetric distribution, symmetric about θ. Show that $E(X) = \theta$ & Median $(X) = \theta$. **[8]**

b) State and prove Jensen's and Markov inequalities. Give their applications. **[8]**

Q3) a) Describe Non-central t-distribution. Give its applications. **[8]**

b) Decompose the following *cdf* $F_X(x)$ into discrete and continuous components. **[8]**

$$F_X(x) = \begin{cases} 0 & ; x < 0 \\ \frac{1}{4} + \frac{x}{4} & ; 0 \le x < 1 \\ \frac{1}{2} + \frac{x}{4} & ; 1 \le x < 2 \\ 1 & ; x \ge 2 \end{cases}$$

Hence, comupte E(X), V(X) and MGF of X.

Q4) a) Let X~V (–2,3). Let $Y = X^{+} = \begin{cases} X & , \text{if } X \ge 0 \\ 0 & , o.w. \end{cases}$

and $Z = X^{-} = \begin{cases} -X & , \text{if } X \le 0 \\ 0 & , o.w. \end{cases}$

Find *cdf* of Y and Z. **[8]**

b) Suppose a projectile is fixed at an angle θ above the earth with a velocity v assuming that θ is a random variable with pdf

$$f(\theta) = \begin{cases} \frac{12}{\pi} & , \frac{\pi}{6} < \theta < \frac{\pi}{4} \\ 0 & , o.w. \end{cases}$$

find the pdf of range R of projectile defined of $R = \frac{v^2 \sin z\theta}{g}$, Where $g =$ gravitational constant. **[8]**

Q5) a) Define a bivariate distribution function. State its properties. Verity whether following bivariate function F defines a bivariate distribution function or not. **[8]**

$$F(x,y)=\begin{cases}0 & ; \quad x<0 \text{ or } x+y<1 \text{ or } y<0 \\ 1 & ; \quad o.w.\end{cases}$$

b) A fair coin is tossed 3 times. Let x = number of heads in 3 tosses and y = absolute difference between number of heads & tails. Find joint and marginal distribution of X and Y. **[8]**

Q6) a) Let a random variable x has pdf. **[8]**

$$F(x)=\begin{cases}\dfrac{1}{\theta_2+\theta_1} & ; \quad -\theta_1<x<\theta_2, \theta_2>-\theta_1 \\ 0 & ; \quad o.w.\end{cases}$$

Find distribution of y = 1 |x|.

b) Let X~ Exp(θ), θ rate, Find distribution of y = [x], where [x] denotes largest integer not larger than X. Compute E(y) and V(y). **[8]**

Q7) Write short notes on the following : **[4 × 4 = 16]**

a) Fisher-Cochran theorem and its applications.

b) Probability Integral transform and its applications.

c) Marshall-Olkin bivariate exponential distribution.

d) Compound distributions.

B – 814

Total No. of Pages : 3

Seat No.	2002

M.Sc. (Part - I) (Semester - I) Examination, October - 2015

APPLIED STATISTICS AND INFORMATICS (Paper - III)

Probability Distributions (CBCS)

28 NOV 2015 **Sub. Code: 61057**

Day and Date : Saturday, 31-10-2015 Total Marks : 80

Time : 10.30 a.m. to 1.30 p.m.

Instructions : 1) Question No. 1 is compulsory.

2) Attempt any four questions from question No. 2 to 7.

3) Figures to right indicate marks to the questions.

Q1) Solve the following sub-questions : [16 × 1 = 16]

a) State Jensen's inequality.

b) Obtain m.g.f. of negative binomial distribution.

c) Define Symmetric family of distributions.

d) Give an example of a non-regular family of distribution.

e) Define bivariate poisson distribution.

f) Define mixture of two distribution functions.

g) Define DFR class of distribution with illustration.

h) Prove or disprove : If X ~ B(n, p) then 2X has Binomial (2n, p) distribution.

i) State the distribution of r^{th} order statistics from an exponential distribution.

j) Find m.g.f. of negative binomial distribution.

k) Examine whether the following function is p.d.f. or not.

$$f(x)=\begin{cases}\sin x & 0<x<\dfrac{\pi}{2}\\ 0 & o.w.\end{cases}$$

P.T.O.

l) If F(X) is a distribution function of a r.v.X, then prove that $[F(X)]^n$ is also a distribution function.

m) Define quantile of a r.v.X.

n) State properties of distribution function.

o) Let X ~ U(0, 1). Obtain distribution of Y = –logX.

p) Define Dirichlet distribution.

Q2) a) Describe the decomposition of a distribution function into discrete and continuous parts. [8]

b) Define bivariate Normal distribution. Find marginal p.d.f's of X and Y. When $(X, Y) \sim BN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. [8]

Q3) a) Obtain joint distribution of r^{th} and s^{th} order statistics. [8]

b) Let $f(x, y) = \frac{1}{y} e^{-y}$, $0 < x < y < \infty$. Find $E(X^r/Y=y)$ where r > 0, is an integer. [8]

Q4) a) State and prove Chebychev's inequality. Verify the same when X ~ exp(1). [8]

b) Decompose the following distribution function in to discrete and continuous components. Obtain m.g.f. and hence find mean and variance

$$F(x) = \begin{cases} 0 & , x < 0 \\ 1 - \frac{e^{-x}}{2} & , x \geq 0 \end{cases}$$ [8]

Q5) a) Define convolution of two random variables. Let X and Y are exponential variates with parameter '1'. Obtain distribution of X + Y. [8]

b) Let $f(x, y) = \frac{3x+y}{4} e^{-x-y}$; x > 0, y > 0. Find correlation coefficient of (X, Y). [8]

Q6) a) Obtain m.g.f. of bivariate poisson distribution.

b) Let X and Y are i.i.d. U(0,1). Obtain distribution of min(X,Y).

c) Let X ~ exp(λ), λ > 0. Obtain the distribution function of $Y = \frac{1}{X}$.

[6+6+4]

Q7) Write short notes : [4 × 4 = 16]

a) Holders inequality.

b) Markov inequality.

c) Location and scale family.

d) Order statistics.

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CBCS E - 1146

Total No. of Pages : 3

Seat No.	

M.Sc. (Part - I) (Semester - I) Examination, Nov. - 2013

APPLIED STATISTICS AND INFORMATICS (Paper - III)

Probability distributions

Sub. Code : 61057

Day and Date : Saturday, 16 - 11 - 2013 Total Marks : 80

Time : 10.30 a.m. to 1.30 p.m.

Instructions : 1) Question no. 1 is compulsory.

2) Attempt any four questions from question no 2 to 7.

3) Figures to right indicate marks to the questions.

Q1) Solve the following sub questions : **[16]**

a) Define a distribution function.

b) Show that $F(x)=\begin{cases}0 & x<0\\ \dfrac{x+1}{2} & 0\le x<1\\ 1 & x\ge 1\end{cases}$, is a cdf.

c) Obtain E(x) where x has cdf as given in (b) above.

d) Define probability generating function (pgf).

e) Obtain pgf for a poisson distribution.

f) Define Dirichilet distribution.

g) State the distribution of r^{th} order statistics from an exponential distribution.

h) Define a symmetric distribution.

i) Give one example of a symmetric distribution.

j) Obtain mgf of a Gamma(a,b) random variable.

k) Let $P[X=x, Y=y]=\frac{1}{2}|x-y|$, x, y = 0, 1. Obtain marginal distributions of X and Y.

P.T.O.

l) Give one example where movement generating function does not exists.

m) If $X \sim N(\mu, \sigma^2)$, find the distribution of $y = e^{-x}$.

n) Let $X \sim \cup(-1, 1)$. Obtain distribution of $[x]$ where $[x]$ denotes the largest integer less than or equal to x.

o) Give one example of a non regular family of distributions.

p) Let F(x) be a distribution function. Examine for what values of α, $F^{\alpha}(x)$ is a distribution function.

Q2) a) Let X has a distribution symmetric about O. Then show that $EX^{2r+1} = 0$ $r = 0, 1, 2, \ldots$

b) The p.m.f. of (X, Y) is given by

$$p(x, y) = \frac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y}}{y!\,(x-y)!}$$

$y = 0, 1, 2, \ldots, x,$

$x = 0, 1, 2, \ldots$

$0 < p < 1, \lambda > 0.$

Obtain marginal and conditional distributions of X and Y.

[8 + 8]

Q3) a) Let a function F be defined by

$$F(x) = \begin{cases} 0 & \text{if } x<0 \\ \dfrac{pr+(1-p)x}{k} & \text{if } r \le x < r+1, \quad r = 0, 1, \ldots, k-1 \\ 1 & \text{if } r \ge k \end{cases}$$

Examine whether F is a distribution function

If yes, decompose it in to continuous and discrete components and identify the component distributions.

b) Obtain E X and V(x) if $x \sim F$.

[8 + 8]

Q4) a) Let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be the order statistics from an exponential distribution with parameter θ.

Let $X_{(0)} = 0$. Show that $Y_i = X_{(i)} - X_{(i-1)}$, $i = 1, \ldots n$ are independent random variables.

b) State and prove Markov inequality.

[8+8]

Q5) a) Define a Marshall - Olkin bivariate exponential distribution (BVE). Let $(X, Y) \sim BVE(\lambda_1, \lambda_2, \lambda_{12})$.

i) Obtain $p[x = Y]$.

ii) Describe a shock model giving rise to BVE.

b) Let $x = (x_1, \ldots, x_k)'$ be a random variable with multinomial distribution with parameters $(n, p_1, \ldots, p_k)$, $\Sigma p_i = 1$. Obtain the variance - Covariance matrix Σ of X. Show that the determinant $|\Sigma| = 0$.

[8+8]

Q6) a) Explain the following terms

i) Convolution of distributions

ii) Mixtures of distributions

iii) Location - scale family of distributions

iv) Forgetfulness property.

b) Give one example each for the terms in (a) above

c) Let X and Y be independent standard normal variates. Obtain the distribution of Z = X, Y.

[6+4+6]

Q7) Write short notes on the following [4 × 4 = 16]

i) ~~Bivariate poisson distribution.~~

ii) Marginal and conditional distributions

iii) Jacobion of transformation

iv) Relation of distribution function with uniform distribution.

CBCS D – 1200

Total No. of Pages : 4

Seat No.

M.Sc. (Part - I) (Semester - I) Examination, March - 2014
APPLIED STATISTICS AND INFORMATICS (Paper - III)
Probability Distributions (New)
Sub. Code : 61057

Day and Date : Saturday, 29 - 03 - 2014 Total Marks : 80

Time : 11.00 a.m. to 2.00 p.m.

Instructions:
1) Question No. 1 is compulsory.
2) Attempt any four questions from Question No.2 to 7.
3) Figures to right indicate marks to the questions.

Q1) Solve the following subquestions [16 × 1]

a) If $X \sim U(-1,2)$ Find pdf of $|X|$.

b) Define moment generating function of a random variable.

c) Define probability mass function of a r.v. X.

d) Examine whether $F(x) = 0$ if $x \leq 0$; $= x$ if $0 \leq x \leq ½$; $= 1$ if $x > ½$ is a distribution function.

e) Define bivariate normal random variable.

f) Define quantile of a r.v. X.

g) Obtain α^{th} quantile of Weibull (a,b) distribution.

h) Obtain mgf of Bernoulli random variable.

i) Obtain pgf of Poisson (λ) distribution.

j) Write down mgf of $y = ax+b$ in terms of mgf of X > namely $M_x(t)$.

k) Find $p[-\infty < X < 2]$ when $X \sim \exp(1)$.

P.T.O.

l) Let $f_i(x)$ be pdf, i=1,2. State conditions under which $g(x)=af_1(x)+bf_2(x)$ is a pdf.

m) Define mixture of two distribution functions.

n) Examine Cauchy distribution for symmetry.

o) Obtain EX^r when X~ exp (λ).

p) State Cauchy - Swartz inequality.

Q2) a) State and prove Jordan decomposition theorem.

b) Let X be an r.v. with d.f.

$$f(x) = 0 \quad , x < 0$$

$$= 1 - pe^{-x/\theta} \quad , x \geq 0$$

Decompose the above d.f. into discrete and continuous parts. Further obtain its mean and variance.

[8 + 8]

Q3) Define symmetric distribution, marginal distribution and conditional distribution.

Let (x,y) have the joint pmf. as follows.

Y	X:1	2	3
1	1/12	2/12	1/12
2	1/12	0	1/12
3	2/12	0	4/12

Show that x and y are identically distributed but not independent. Find conditional distribution of X given Y=1.

Find distribution of Z = x-y and show that it is symmetric. [16]

Q4) a) State Markov's inequality. Show that Chebychev's inequality is a particular case of Markov's inequality. Verify both the inequalities for exp (1) distribution.

b) Let X be an r.v. with pdf,

$$f_\theta(x) = \begin{cases} \theta e^{-\theta x}, & x \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

Let $Y = \left[x - \frac{1}{\theta}\right]^2$. Find the pdf of Y.

[8 + 8]

Q5) Define order statistics. Write down the joint pdf of $x_{(1)}, \ldots, x_{(n)}$, n order statistics from a continuous distribution.

Let $x_{(1)}, \ldots, x_{(n)}$ be the order statistics of n independent r.v's $x_1 \ldots x_n$ with common pdf $f(x)=1$ if $0 < x < 1$ and $= 0$ otherwise. Show that $Y_1 = X_{(1)}/X_{(2)}$, $Y_2 = X_{(2)}/X_{(3)}$, ..., $Y_{n-1} = X_{(n-1)}/X_{(n)}$ and $Y_n = X_{(n)}$ are independent. Find the pdf's of $Y_1, Y_2, \ldots, Y_n$.

[16]

Q6) a) Define convolution of two random variables illustrate with a suitable example.

b) State Jensen's inequality. Give one application.

c) Define bivariate poisson distribution. Obtain its mgf.

[5 + 5 + 6]

Q7) Write short notes

a) Bivariate Marshall - Olkin distribution

b) Characteristic function of a r.v.

c) Exponential spacings

d) Dirichlet distribution

[4 × 4]

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W – 447

Seat No.

M.Sc. (Part – I) (Semester – I) Examination, 2012
(Credit System)
STATISTICS (Paper – III)
Distribution Theory
Sub. Code : 42323

Day and Date : Monday, 29-10-2012 Total Marks : 80

Time :10.30 a.m. to 1.30 p.m.

Instructions: 1) Question 1 is compulsory.

2) Attempt any 4 questions from Q.2 to Q.7.

3) Figures to the right indicate full marks to the sub-question.

1. Attempt any eight questions.

a) Define distribution function of a random variable.

b) Let X follows binomial distribution with parameters n and p. Find the probability mass function of $Y = aX+b$ where a and b are constants.

c) Check whether following f (x) is a p.d.f.

$f(x) = \sin x, \quad 0 < x < \pi/2$

$= 0, \quad$ otherwise

d) Check whether following F (x) is a distribution function. If yes, find the corresponding p.d.f.

$F(x) = 0$ for $x \leq 0$, $= \frac{x}{2}$ for $0 \leq x < 1$, $= \frac{1}{2}$ for $1 \leq x < 2$, $= \frac{x}{4}$ for $2 \leq x < 4$ and $= 1$ for $x \geq 4$.

P.T.O.

e) Define a probability generating function and give one example.

f) Suppose $P(X \geq x) = 1$ if $x \leq 0$ and
$= e^{-\lambda x}$ for $x > 0,\ \lambda > 0$
then find the pdf of the r.v. X.

g) State Markov's inequality and Chebychev's inequality.

h) Let X_1, X_2 be independent binomial $X_i \sim B\left(n_i, \frac{1}{2}\right)$ random variables $i = 1, 2$. What is the p.m.f. of $X_1 - X_2 + n_2$?

i) Define moment generating function of a r.v. State one example of it for a discrete r.v.

j) A fair coin is tossed three times. Let X = number of heads in three tossings and Y = absolute difference between number of heads and number of tails. Obtain the joint p.m.f. of (X, Y). (2×8)

2. a) Let X and Y be independent geometric r.vs. Show that min (X, Y) follows geometric distribution.

b) Let X be an r.v. with p.d.f.

$$f(x) = \begin{cases} \theta e^{-\theta x}, & x \geq 0, \\ 0, & \text{otherwise where } \theta > 0. \end{cases}$$

Let $Y = \left(X - \frac{1}{\theta}\right)^2$. Find the pdf of Y.

c) Let X be an r.v. with pdf. $f(x) = \frac{1}{2\pi}$; $0 < x < 2\pi$,
$= 0$; otherwise

Let Y = sin X find the distribution function and pdf of Y. (5+6+5)

3. a)

b)

4. a

b

5.

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Impr. Cr-W – 1001

Seat No.

M.Sc. (Part – I) (Semester – I) Examination, 2011
STATISTICS (Paper – III) (Credit System)
Distribution Theory

Day and Date : Friday, 29-4-2011
Time : 11.00 a.m. to 2.00 p.m.

Total Marks : 80

Instructions : a) Question No.1 is compulsory.
b) Attempt any 4 questions from No. 2 to 7.
c) Figures to right indicate marks to the sub question.

1. Attempt any **eight** sub-questions :

a) Describe an application of lognormal distribution.

b) Obtain the lower quartile of two-parameter Cauchy distribution.

c) Obtain the mean and variance of Y where $P(Y = y) = q^y p$, $y = 0, 1, 2, \ldots$ and $p+q = 1$.

d) Obtain an expression for mgf of multinomial distribution.

e) If $X_1, \ldots X_n \sim U(0, 1)$ find the distribution of $X_{(1)}$.

f) Define characteristic function. State its important properties.

g) Define distribution function of a random variable.

h) If $X_1, X_2, X_3, \sim N(0, 1)$, state the distribution of $\frac{2x_1^2}{X_2^2 + X_3^2}$ with justification.

i) Define joint and conditional pmf of a bivariate distribution.

j) Suppose the r.vs. X and Y have joint pdf

$$f(x, y) = \begin{cases} kx(x-y) & 0 < x < 2; -x < y < 2 \\ 0 & \text{o.w.} \end{cases}$$, evaluate k.

P.T.O.

2. a) Define mixture of two distribution functions. Examine the same for being a distribution function.

b) Prove that, every non negative real function f that is integrable over R and satisfies $\int_{-\infty}^{\infty} f(x)dx = 1$ is the pdf of some continuous r.v. X.

c) Let $f(x) = \begin{cases} x & 0 < x \le 1 \\ 2-x & 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$. Sketch the graph of f(x). Also obtain the d.f. F(x) of the given pdf and sketch the same. (4+4+8)

3. a) State and prove Holder's inequality.

b) Define convolution of two distribution functions F_1 and F_2. Obtain the same when F_1 and F_2 are standard exponential distribution functions. (8+8)

4. a) Explain and illustrate decomposition of a distribution function into discrete and continuous components.

b) If X and Y are independent Poisson r.vs. with means θ_1 and θ_2 respectively, find the conditional distribution X given x + y. (8+8)

5. a) Define bivariate normal distribution $BN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Obtain conditional distribution of X given Y when (X, Y)' has $BN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Also obtain mean of the conditional distribution.

b) Define location and scale families. Illustrate with examples. (12+4)

6. a) Reduce the quadratic form

$6x_1^2+35x_2^2+11x_3^2+34x_2x_3$ into canonical form.

b) If $\underline{X}'A\underline{X}$ is a quadratic of rank r, then show that there exists an orthogonal transformation $\underline{X}=P\underline{Y}$ such that $\underline{X}'A\underline{X}=\sum_{i=1}^{r}\lambda_i y_i^2$, where $\lambda_1, \lambda_2, ..., \lambda_r$ are characteristic roots of A. (8+8)

7. Write short notes on any **four** of the following :

a) Inverse of a partitioned matrix.

b) Particular and general solution of $A\underline{X}=\underline{b}$.

c) Classification of quadratic forms.

d) Homogenous system of equations.

e) Left & right inverse of a matrix.

f) Orthogonal matrix. (4×4)

Cr.G – 684

Total No. of Pages : 3

Seat No.	

M.Sc.(Part - I) (Semester - I) Examination, 2013
STATISTICS (Paper - III) (A.F.) (Credit System)
Distribution Theory
Sub. Code : 42323

Day and Date : Tuesday , 23 - 04 - 2013 **Total Marks : 80**

Time : 3.00 p.m. to 6.00 p.m.

Instructions : 1) Question No. 1 is compulsory.

2) Attempt any 4 questions from No. 2 to 7.

3) Figures to right indicate marks to the sub-question.

Q1) Solve any eight sub questions. [8×2]

i) Let $X \sim U(-1,1)$. Obtain distribution function of $Y = |X|$

ii) Examine whether $F(x)=x^2$ $0 \le x \le 1$; Zero otherwise is a distribution function.

iii) Define Probability generating function. Obtain the same for Poisson (λ) distribution.

iv) State an example where m.g.f. is not well defined in an interval around $t = 0$

v) Comment on the Statement : characteristic function always exists.

vi) Let X be any continuous random variable with cumulative distribution function $F(x)$. Obtain the distribution of $Y= -\log \bar{F}(x)$ Where $\bar{F}(x) = 1-F(x)$.

vii) Prove or disprove : If $X \sim$ Binomial (n,p) then 2X has Binomial (2n,p) distribution.

viii) Define quantile of a probability distribution . Give one example.

ix) Define bivariate Marshall - olkin distribution

P.T.O.

Q2) a) State and prove relation between distribution function of a continuous random variable and uniform random variable.

b) Describe applications of above relationship in random number generation.

c) Show that for a non -negative continuous random variable X with distribution function $F(x)$,

$$E(X) = \int_0^{\infty} (1-F(u))\, du$$

[6 + 5 + 5]

Q3) a) Describe a procedure for decomposing a distribution function in to discrete and continuous components.

b) Let $U \sim U(0,1)$ Define a random variable X as

$$X = \begin{cases} 0.5 & \text{if} \quad U < 0.3 \\ 1 & \text{if} \quad 0.3 \le U < 0.8 \\ 1.5 & \text{otherwise} \end{cases}$$

obtain distribution of X. Obtain its mean and variance.

[6 + 10]

Q4) a) State and prove Tcheby cheff's inequality. Varify the same when $X \sim \exp(1)$.

b) State and prove use of probability generating function for obtaining factorial moments.

[10 + 6]

Q5) a) Define convolution of two random Variables. Obtain the same when X_1 and X_2 are i i d exp (1).

b) Define location family, scale family and location -scale family of distributions. Give one example of each.

[8 + 8]

Q6) a) Define and illustrate the following terms

i) moment generating function

ii) symmetric distribution.

iii) marginal and conditional distributions

iv) order statistics.

b) Obtain the joint distribution of $(i,j)^{th}$ order statistics for U(0,1) distribution.

[12 + 4]

[4 × 4]

Q7) Write short notes on any four

a) multinomial distribution

b) Jacobian of transformation.

c) Jensen inequality

d) Exponential spacings

e) Bivariate exponential distribution.

f) Joint distributions with given marginals

Impr. Cr-W – 1001

Seat No.

M.Sc. (Part – I) (Semester – I) Examination, 2011
STATISTICS (Paper – III) (Credit System)
Distribution Theory

Total Marks : 80

Day and Date : Friday, 29-4-2011

Time : 11.00 a.m. to 2.00 p.m.

Instructions : a) Question No.1 is compulsory.

b) Attempt any 4 questions from No. 2 to 7.

c) Figures to right indicate marks to the sub question.

1. Attempt any **eight** sub-questions :

a) Describe an application of lognormal distribution.

b) Obtain the lower quartile of two-parameter Cauchy distribution.

c) Obtain the mean and variance of Y where $P(Y = y) = q^y p$, $y = 0, 1, 2, \ldots$ and $p+q = 1$.

d) Obtain an expression for mgf of multinomial distribution.

e) If $X_1, \ldots X_n \sim U(0, 1)$ find the distribution of $X_{(1)}$.

f) Define characteristic function. State its important properties.

g) Define distribution function of a random variable.

h) If $X_1, X_2, X_3, \sim N(0, 1)$, state the distribution of $\dfrac{2X_1^2}{X_2^2 + X_3^2}$ with justification.

i) Define joint and conditional pmf of a bivariate distribution.

j) Suppose the r.vs. X and Y have joint pdf

$$f(x, y) = \begin{cases} kx(x-y) & 0 < x < 2; -x < y < 2 \\ 0 & \text{o.w.} \end{cases}$$, evaluate k.

P.T.O.

2. a) Define mixture of two distribution functions. Examine the same for being a distribution function.

b) Prove that, every non negative real function f that is integrable over R and satisfies $\int_{-\infty}^{\infty} f(x)dx = 1$ is the pdf of some continuous r.v. X.

c) Let $f(x) = \begin{cases} x & 0 < x \leq 1 \\ 2-x & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$. Sketch the graph of f(x). Also obtain the d.f. F(x) of the given pdf and sketch the same. (4+4+8)

3. a) State and prove Holder's inequality.

b) Define convolution of two distribution functions F_1 and F_2. Obtain the same when F_1 and F_2 are standard exponential distribution functions. (8+8)

4. a) Explain and illustrate decomposition of a distribution function into discrete and continuous components.

b) If X and Y are independent Poisson r.vs. with means θ_1 and θ_2 respectively, find the conditional distribution X given x + y. (8+8)

5. a) Define bivariate normal distribution $BN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Obtain conditional distribution of X given Y when (X, Y)' has $BN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Also obtain mean of the conditional distribution.

b) Define location and scale families. Illustrate with examples. (12+4)

6. a) Reduce the quadratic form

$6x_1^2 + 35x_2^2 + 11x_3^2 + 34x_2x_3$ into canonical form.

b) If $\underline{X}'A\underline{X}$ is a quadratic of rank r, then show that there exists an orthogonal transformation $\underline{X} = P\underline{Y}$ such that $\underline{X}'A\underline{X} = \sum_{i=1}^{r} \lambda_i y_i^2$, where $\lambda_1, \lambda_2, ..., \lambda_r$ are characteristic roots of A. (8+8)

7. Write short notes on any **four** of the following :

a) Inverse of a partitioned matrix.

b) Particular and general solution of $A\underline{X} = \underline{b}$.

c) Classification of quadratic forms.

d) Homogenous system of equations.

e) Left & right inverse of a matrix.

f) Orthogonal matrix. (4×4)

Cr-F – 198

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2011
(Credit System)
STATISTICS (Paper – III)
(Distribution Theory)
Sub. Code : 42323

Day and Date : Monday, 17-10-2011
Time : 10.30 a.m. to 1.30 p.m.

Total Marks : 80

Instructions : 1) *Question No. 1 is compulsory.*
2) *Attempt any 4 questions from No. 2 to 7.*
3) *Figures to the right indicate marks of the sub questions.*

1. Solve any eight sub-questions :

1) Let X~ exp (λ), $\lambda > 0$. Obtain the distribution function of $Y = \frac{1}{X}$.

2) Let X~ Bernoulli (P), $0 < P < 1$. Let Y_1 and Y_2 are independent standard exponential random variables.

Let $Z = Y_1$ if $X = 0$

$= Y_2$ if $X = 1$

Obtain Ez.

3) Define convolution of two distribution functions.

4) Define moment generating function (mgf). Give an example where mgf does not exist.

5) State Morkov inequality.

6) Let (X, Y) have joint p.m.f. given by $P(x,y) = \frac{1}{2e} \frac{e^{-1} + 2^{-(x+y)}}{x!\,y!}$ x, y = 0, 1 . . .

Obtain marginal p.m.f. of X and of Y.

P.T.O.

7) State sufficient conditions under which (X, Y) are independently distributed.

8) Define order statistic. Obtain the distribution of first order statistic from a random sample from U (0, 1) distribution.

9) Let f be a density function. Examine whether $\alpha f(\alpha x), \alpha > 0$ is a density function.

10) Define a symmetric distribution. Give one example of the same. (2×8)

2. a) Define a distribution function. Let F_1 and F_2 be two distribution functions. Examine whether the following functions are distribution functions.

i) $F(x) = \alpha F_1(x) + (1-\alpha) F_2(x) \quad 0 \le \alpha \le 1, x \in \mathbb{R}$

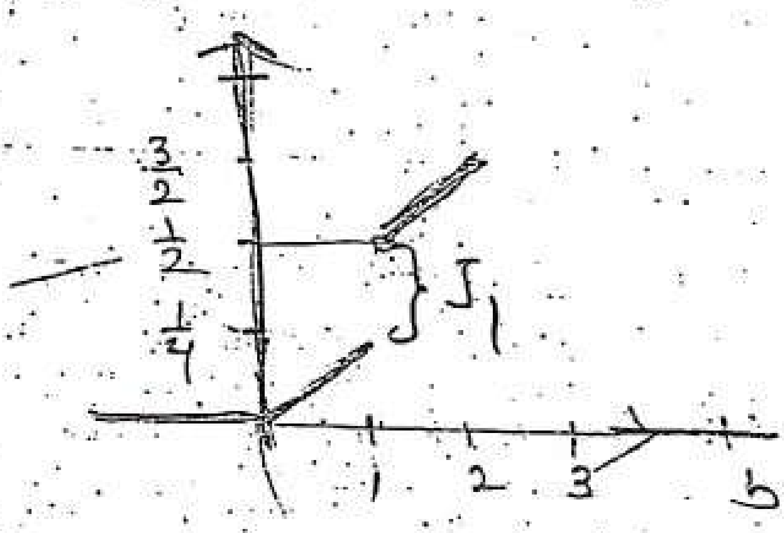
ii) $F(x) = F_1(x) F_2(x) \quad x \in \mathbb{R}$

iii) $F(x) = 1 - F_1(-x) \quad x \in \mathbb{R}$

b) Let F(x) be the distribution function of a continuous random variable X. Obtain the distribution of $H(X) = -\log(1-F(x))$. (4×3+4)

3. a) Decompose the following distribution function in to discrete and continuous components.

$$F(x) = \begin{cases} 0 & x < 0 \\ x/4 & 0 \le x < 1 \\ \dfrac{x-1}{4} + \dfrac{1}{2} & 1 \le x < 2 \\ \dfrac{11}{12} & 2 \le x < 3 \\ 1 & 3 < x \end{cases}$$



b) State and prove Jensen's inequality. (8+8)

4. a) Define conditional expectation. Let (X, Y) denote a two dimensional random vector with density function

$$f(x, y) = \frac{1}{y} e^{-y} \quad 0 < x < y < \infty.$$

Find $E(X^r \mid Y = y)$ where r > 0, an integer.

b) State and prove two important properties of order statistics from an exponential distribution. (8+8)

5. a) Let (X, Y) have the joint pdf given by $f(x,y)=\frac{1}{x^3 y^2}$ $x>1, y>\frac{1}{x}$.

   Find the density functions of the marginal distributions of X and Y. Also find CoV (X,Y).

   b) Obtain convolution of two independent exp (θ) random variables. (10+6)

6. a) State Chebychev's inequality and Cauchy Schwartz inequality. Discuss one application of each inequality.

   b) Let X~ exp(θ) and Y = [X] where [a] denotes largest integer less than X. Obtain distribution of Y. State an important application of this result. (8+8)

7. Write short notes on **any four** :

   a) Bivariate distribution function

   b) Transformation of bivariate random variables

   c) Probability generating function

   d) Bivariate Poisson distribution

   e) Dirichlit distribution

   f) Marshall-Olkin distribution. (4×4)

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Total No. of Pages : 3

Seat No. 01957

M.Sc. (Part - I) (Semester - I) Examination, November - 2014
APPLIED STATISTICS AND INFORMATION (Paper - III)
Probability Distributions (CBCS)
Sub. Code : 61057

Day and Date : Friday, 14 - 11 - 2014 **Total Marks : 80**

Time : 10.30 a.m to 1.30 p.m.

Instructions : 1) Question No. 1 is compulsory.

2) Attempt any four questions from Question No. 2 to 7.

3) Figures to the right indicate marks to the questions.

Q1) Solve the following sub - questions : [16 × 1 = 16]

a) Let $X \sim N(\mu, \sigma^2)$. Obtain the distribution of e^X.

b) Let X be a r.v. whose distribution is symmetric about zero. Then obtain E(x).

c) Let X_i, i = 1....n be iid weibull (1, θ). State pdf of $x_{(1)} = \min\{x_1, ..., x_n\}$

d) State minkowski inequality.

e) State one application of minkowasky inequality.

f) Let $x_1, ..., x_n$ be iid multinomial $(n, p_1, ..., p_k)$ obtain $Cov(x_i, x_j)$ $i \neq j$.

g) Let $x \sim f(x) = \begin{cases} 2(1-x) & 0 < x < 1 \\ 0 & otherwise \end{cases}$ obtain mgf of x.

h) Define bivariate poisson distribution.

i) Obtain first order raw moments of poisson distribution.

j) If x_1, x_2 are independent NB (r_1, θ) and NB (r_2, θ) respectively, state the distribution of $x_1 + x_2$.

k) State characterizing properties of a distribution function.

P.T.O

l) Let f(x) be a distribution function. State with reasons whether $[F(x)]^{\alpha}$, $\alpha < 0$ is also a distribution function.

m) Give one example of a skew distribution.

n) Define mixture of two distributions.

o) Define joint moment generating function.

p) State conditions under which two random variables X and Y are independently distributed.

Q2) a) Let U be a uniform r.v. over (0, 1). define a random variable X as. [8]

$$X = \begin{cases} 1 & is \quad U < .4 \\ 2 & if \quad .4 \le U < .9 \\ 0 & otherwise \end{cases}$$

obtain distribution of X. Also obtain its mean and variance.

b) If X_1, X_2 are independent discrete uniform random variables over {0, 1, 2, 3}. Find the pmf of $Y = X_1 + X_2$. Also find E(Y) and V(Y). [8]

Q3) a) State and prove the relation between a uniform random variables and the distribution function of a continuous random variable. Using this result state a procedure for simulating a random sample from [8]

i) Exponential

ii) Weibull distribution

b) Let X ~ exp (λ) distribution and Y = [X] i.e. greatest integer less than or equal to X. Obtain distribution of Y. [8]

Q4) a) Define order statistics. Obtain distribution of sample range when a sample of size n from U(0, θ) is available. [8]

b) State and prove Jensen's inequality. [8]

Q5) a) Define bivariate normal distribution $BN(\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,\rho)$ and obtain conditional distribution of X given Y = y, when (X, Y) have joint $BN(\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,\rho)$ distribution. Examine under what conditions X and Y are independently distributed. [8]

b) Let X and Y be independent N(0, 1) random variables. Define [8]

$$Z=\begin{cases} X & if\ XY>0 \\ -X & if\ XY\le 0 \end{cases}$$

obtain distribution of Z-Examine whether (Z, Y) have a bivariate normal distribution. If yes, obtain the parameters.

Q6) a) Define the following terms and

i) Symmetric distribution

ii) Probability generating function

iii) Convolution of random variables.

b) Give one example each for the terms in (a) above.

c) Outline a procedure for decomposing a given cdf F as a mixture of a discrete and a continuous distribution functions.

[4 + 6 + 6 = 16]

Q7) Write short notes on the following. [4 × 4 = 16]

a) Jacobian of transformation.

b) Location and scale family.

c) Dirichi let distribution.

d) Order statistics from exponential distribution.

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