SLR-ES-373

Seat No.	

Set P

M.Sc. (Semester - II) (CBCS) Examination March/April-2019
Statistics
THEORY OF TESTING OF HYPOTHESIS

Day & Date: Wednesday, 24-04-2019 Max. Marks: 70
Time: 12:00 PM to 02:30 PM

Instructions: 1) All Questions carry equal marks.
2) Figures to the right indicate full marks.

Q.1 Select correct alternatives:- **14**

1) For testing simple null against simple alternative which of the following statement is most appropriate?
   a) UMP level $\propto$ test exists b) UMPU level $\propto$ test exists
   c) MP level $\propto$ test exists d) UMP invariant test exists

2) Let x has $N(\mu, \sigma^2)$ distribution. Consider testing of $H_0 : \mu = 0$ against $H_1 : \mu = 1$. Then _______.
   a) Both H_0 and H_1 are composite b) Both H_0 and H_1 are simple
   c) H_0 is simple and H_1 is composite d) H_0 is composite and H_1 is simple

3) If $\propto$ and β are probabilities of type I and type II errors respectively. Which of the following inequality is satisfied by MP test?
   a) $\propto < \beta$ b) $\propto > \beta$
   c) $\propto + \beta > 1$ d) $\propto + \beta \leq 1$

4) For LRT, asymptotic distribution of -2 log λ is ______.
   a) Normal b) Chi-Square
   c) Gamma d) t

5) For simple against simple hypothesis, MP test and LRT are ______.
   a) equivalent in size but not with respect to power
   b) different
   c) equivalent
   d) not comparable

6) The acceptance region of _______ test leads to UMAU confidence set.
   a) MP b) UMP
   c) UMPU d) Unbiased

7) Generalized N-P lemma is used to construct _________ tests.
   a) MP b) UMP
   c) Unbiased d) None of these

8) Which one of the following is second kind error in testing of hypothesis?
   a) accept H_0 b) reject H_0
   c) accept H_0 when it is false d) reject H_0 when it is true

9) Let $X \sim N(0, \sigma^2)$, $\sigma^2 > 0$. The family of distribution of X has MLR in ______.
   a) X b) X^2
   c) –X d) |X|

10) If $\phi(x) \equiv \propto$ for all x then _______.
   a) $\phi(x)$ is MP test b) $\phi(x)$ is not valid test function
   c) power of $\phi(x)$ is $\propto$ d) $\phi(x)$ is biased test

11) If $\phi_1(x)$ and $\phi_2(x)$ are two tests of size $\propto$ each then size of the test $\lambda\,\phi_1(x) + (1-\lambda)\phi_2(x)$ is _______.
    a) $\propto$
    b) $\lambda \propto$
    c) 1
    d) Not defined

12) Let x ~ U(0,θ) then based o single observation for testing H_0:θ =1 against H_1:θ ≠1 _______.
    a) no test exists
    b) every test that exist is biased
    c) UMP test exists which is not UMP
    d) UMP test exists

13) A test ϕ is said to be $\propto$-similar on a subset $\Theta^* \subset \Theta$ if ________.
    a) $\beta_\phi(\theta) > \propto, \forall\, \theta \in \Theta^*$
    b) $\beta_\phi(\theta) = \propto, \forall\, \theta \in \Theta^*$
    c) $\beta_\phi(\theta) < \propto, \forall\, \theta \in \Theta^*$
    d) None of these

14) Degrees of freedom for χ^2 statistic in case of contingency table of order (3 x 4) is _________.
    a) 12
    b) 9
    c) 6
    d) 3

Q.2 a) Answer the following. (Any four) **08**
1) Define similar test and test with Neyman structure.
2) Define size of test and power of test.
3) Define non-randomized test. Give an example.
4) Show that family of Cauchy densities does not possess MLR property.
5) State Neyman-Pearson lemma.

b) Write short notes on any two of the following:- **06**
1) One sample U statistic
2) Shortest length confidence interval
3) Unbiased test.

Q.3 a) Attempt any two of the following:- **08**
1) Show that power of MP test is greater than $\propto$, if the size of the test is $\propto$.
2) Explain the run test to test randomness.
3) Based on random sample of size n from N (θ,1) distribution, obtain UMP level $\propto$ for testing H_0: $\theta \le \theta_0$ against H_1 : $\theta > \theta_0$

b) Attempt any one of the following:- **06**
1) Let x_1, x_2, -----, x_n be a random sample from pareto population with p.d.f. $f(x,\theta) = \frac{\theta}{x^2}$, $x \ge \theta$. Consider the pivot $Q = \frac{\theta}{x_{(1)}}$. Show that (1- $\propto$) level shortest length confidence interval based on pivot Q is $\left(x_{(1)} \propto^{\frac{1}{n}}, x_{(1)}\right)$
2) Prove that a test with Neyman structure is a similar test. Is the converse true?

Q.4 a) Attempt any two of the following:- **10**
1) Define UMA confidence intervals. How are they obtained from UMP tests.
2) Show that no UMP test exists for testing $H_0: \theta = \theta_0$ against $H_0: \theta \neq \theta_0$ in one parameter exponential family of distributions.
3) A sample of size one is taken from Poisson (λ). For testing against $H_0: \lambda = 1$ against $H_1: \lambda = 2$, the test function is $\phi(x) = \begin{cases} 1, & if\ x > 3 \\ 0, & \text{otherwise} \end{cases}$
Find probability of type I error and power of test.

b) Attempt any one of the following:- **04**

1) Define likelihood ratio test (LRT). Show that LRT for testing simple hypothesis against simple alternative is equivalent to MP test.
2) Describe Chi-square test for goodness of fit.

Q.5 Attempt an two of the following:- **14**

a) Let x_1, x_2, ------, x_n be a random sample from B (1,θ). Obtain MP level ∝ test for testing $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1 > \theta_0$

b) Define UMPU test. Show that every UMP test is UMPU of same size.

c) Describe Wilcoxon signed-rank test for one sample problem.

SLR-MB – 621

Seat No.	

M.Sc. (Part – I) (Semester – II) (Old CGPA) Examination, 2016
STATISTICS (Paper – IX)
Theory of Testing of Hypotheses

Day and Date : Wednesday, 6-4-2016 Total Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*

*2) Q. No. **(1)** and Q. No. **(2)** are **compulsory**.*

*3) Attempt **any three** from Q. No. **(3)** to Q. No. **(7)***

*4) Figures to the **right** indicate **full** marks.*

1. A) Choose correct alternative from the given alternatives.

1) Size of a test is

a) Always greater than or equal to the level of significance

b) Always less than or equal to the level of significance

c) Always equal to the level of significance

d) Some times greater than the level of significance

2) Reject H_0, when it is true

a) Type I error b) Type II error

c) Probability of type I error d) Probability of type II error

3) A non-randomized test function takes values

a) –1 or + 1 b) in (0, 1) c) 0 or 1 d) none of these

4) If $\lambda(x)$ denotes likelihood ratio statistic, then the asymptotic distribution of $-2 \log \lambda(x)$, under certain regularity condition is

a) Chi-square b) Exponential c) Uniform d) Normal

5) For testing $H_0 : \theta = 1$ against $H_1 : \theta \neq 1$ based on a random sample from $N(\theta, 1)$

a) MP test exists b) UMP test does not exist

c) Both (a) and (b) d) None of these

P.T.O.

B) Fill in the blanks.

1) UMP test leads to ____________ confidence interval.

2) Generalised NP lemma is used to construct ____________ test.

3) The degrees of freedom associated with m × n contingency table is ___________.

4) The UMP test in the class of unbiased test is known as _____________.

5) One parameter exponential family of distribution has ____________ property.

C) State whether the following statements are **True** or **False**.

1) The family of Cauchy distributions possesses an mlr property.

2) An MP test has power less than its size.

3) A test function $\phi(x) = 0.5$ for all x has power 0.5.

4) If ϕ_1 and ϕ_2 are two test function then $2\phi_1 + \phi_2$ is a test function. **(5+5+4)**

2. a) Explain the term :

1) U-statistic

2) Monotone likelihood ratio property.

b) Write short notes on the following :

1) Similar test

2) Neyman Pearson lemma. **(6+8)**

3. a) Explain the terms :

I) Randomised test

II) Non-randomised test

III) Power function

Give one example for each.

b) Obtain MP test of size 0.2 for testing $H_0 : f = f_0$ against $H_1 : f = f_1$

X :	1	2	3	4
$\mathbf{f_0}$:	0.1	0.2	0.2	0.5
$\mathbf{f_1}$:	0.35	0.3	0.3	0.05

1) Compute the power.

2) Is MP test unique ? Justify. **(7+7)**

4. a) Let X have density f (x, θ), $\theta \in \mathbb{R}$ and the families of densities have mlr property. Derive UMP test for $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$

   b) Let $X_1, X_2, \ldots, X_n$ be iid rvs with N (0, σ^2) distribution. Obtain UMP test of size α for testing $H_0 : \sigma^2 \leq 1$ against $H_1 : \sigma^2 > 1$. **(7+7)**

5. a) Explain the terms :

      1) UMP test

      2) UMPU test

      3) Test with Neyman structure.

   b) Obtain UMPU size α test for testing $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \neq \sigma_0^2$ based on a random sample of size n from N (0, σ^2) distribution. **(7+7)**

6. a) Describe LRT procedure for testing $H_0 : \theta \in \Theta_0$ against $H_1 : \theta \in \Theta_1$. Show that LRT for testing simple hypothesis against simple alternative is equivalent to M.P. test.

   b) Let $X_1, X_2, \ldots, X_n$ be a random sample from N (μ, σ^2), σ is unknown. Find the (1 – α) level confidence interval for μ by pivotal method. **(7+7)**

7. a) Describe Wilcoxon signed rank test.

   b) Explain :

      1) Shortest length confidence interval

      2) Chi-square goodness of fit test. **(7+7)**

SLR-JS-375

Seat No.	

Set P

M.Sc. (Semester - II) (CBCS) Examination Oct/Nov-2019
Statistics
THEORY OF TESTING OF HYPOTHESES

Day & Date: Friday, 08-11-2019 Max. Marks: 70
Time: 11:30 AM To 02:00 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. **14**

1) If $\propto$ and β are probability of Type I and Type II errors. Which one of the following is the probability of rejecting H_0 when H_1 is true?
   a) $\propto$ b) $1-\propto$
   c) β d) $1-\beta$

2) The p.d.f. $f(x) = \frac{1}{2}e^{-|x-\theta|}$, $-\infty < x < \infty$, has MLR in _________.
   a) x^2 b) $|x|$
   c) x d) $-x$

3) For comparing two test functions, which of the following measure is appropriate?
   a) Size of test
   b) Power of test
   c) Variance of underlying test statistic
   d) Unbiasedness of the test statistic

4) For goodness of fit test, the value of χ^2 statistic is zero if and only if ____.
   a) $\sum O_i = \sum E_i$ b) $\sum O_i^2 = \sum E_i^2$
   c) $O_i = E_i$ for all i d) None of these

5) Test with Neyman structure is a _________.
   a) Similar test b) Subset of similar tests
   c) Not a subset of similar tests d) None of these

6) On the basis of single observantion X from $U(0, \theta)$ distribution, the critical region for testing $H_0: \theta = 1$ against $H_1: \theta = 2$ is defined as $\{0.5 < X < 2\}$. Then power if the test is __________.
   a) 0.25 b) 0.50
   c) 0.75 d) 0.90

7) Let $X_1, X_2, \ldots, X_n$ be iid $N(\theta, 1)$. Let $H_0: \theta = \theta_0$ and $H_1: \theta \neq \theta_0$. The UMPU level $\propto$ test rejects H_0 iff __________.
   a) $\bar{X} > C_1$ b) $\bar{X} < C_2$
   c) $C_1 < \bar{X} < C_2$ d) $\bar{X} < C_1$ or $\bar{X} > C_2$

8) A test function $\emptyset(x) \equiv 0.5$ for all x, has power __________.
   a) 1 b) 0
   c) 0.5 d) None of these

9) Let $H_1: \mu = 5$, where μ is mean of normal population from which sample is taken.
H_2 : population follows standard normal distribution.
a) H_1 is simple and H_2 is simple
b) H_1 is simple and H_2 is composite
c) H_1 is composite and H_2 is simple
d) H_1 is composite and H_2 is composite

10) A family of $U(0, \theta)$ distribution has MLR in _________ when sample of size n is available from $U(0, \theta)$.
a) $\bar{X}$
b) $X_{(1)}$
c) $X_{(n)}$
d) None of these

11) A test for testing H_0 against H_1 is called level $\propto$ test if _________.
a) Size of test does not exceeds $\propto$
b) Size of test is exactly equal to $\propto$
c) Hypothesis of the test is simple hypothesis
d) The test is unbiased

12) For $N(\theta, 1)$ distribution, pivotal quantity for confidence interval of θ based on $X_1, X_2, \ldots, X_n$ is __________.
a) $n\,\bar{X}$
b) $\sqrt{n}\,\bar{X}$
c) $n\,(\bar{X} - \theta)$
d) $\sqrt{n}\,(\bar{X} - \theta)$

13) A UMP test is __________.
a) Always exists
b) Biased test
c) Unbiased test
d) None of these

14) The acceptance region of UMP size $\propto$ test leads to ______ confidence set.
a) UMA
b) UMAU
c) Biased
d) Unbiased

Q.2 A) Answer the following questions. (Any Four) **08**

1) Define simple hypothesis and composite hypothesis. Give one example for each.
2) Define pivotal quantity. Give an example.
3) Define U statistic and give an example.
4) Define UMA confidence interval.
5) Define likelihood ratio test.

B) Answer the following questions. (Any Two) **06**

1) Test for independence of attributes.
2) Mann-Whitney test
3) Type I and type II errors

Q.3 A) Answer the following questions. (Any Two) **08**

1) Define monotone likelihood ratio (MLR) of probability distributions. Show that exponential distribution with mean θ possess MLR property.
2) Prove or disprove: MP test is not unique
3) Use N-P lemma to test $H_0: \theta = 0$ against $H_1: \theta = 1$ on the basis of random sample of size n from $N(\theta, 1)$ distribution.

B) Answer the following questions. (Any One) **06**

1) Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(0, \theta)$ distribution. Obtain $(1-\propto)$ level shortest length confidence interval for θ.
2) Explain the concept of unbiased test. Examine whether MP test is necessarily unbiased.

Q.4 A) Answer the following questions. (Any Two) **10**

1) State and prove a necessary condition under which a UMP size $\propto$ similar test is UMPU test.
2) Derive the relationship between UMA confidence set and UMP test.
3) Let $X_1, X_2, \dots., X_n$ are iid $N(\theta, \sigma^2)$, where σ^2 is known. Show that UMP test does not exists for testing $H_0: \theta = \theta_0$ and $H_1: \theta \neq \theta_0$

B) Answer the following questions. (Any One) **04**

1) Derive LRT for testing $H_0: \theta = \theta_0$ and $H_1: \theta \neq \theta_0$ based on a sample of size n from $N(\theta, 1)$ distribution.
2) State the generalized Neyman-Pearson lemma. Also explain in detail any one of its application.

Q.5 Attempt any two of the following questions. (Any Two) **14**

1) Obtain the UMPU level $\propto$ test for testing $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$ based on $N(\theta, \sigma^2)$, where σ^2 is known for a sample of size n.
2) Describe χ^2 test for goodness of fit.
3) Let $X \sim B(6, \theta)$. $H_0: \theta = \frac{1}{2}$, $H_1: \theta = \frac{3}{4}$. Compute the probabilities of type I and type II errors when test is given by reject H_0 if $X = 0, 6$.

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – IX) (Old)
Theory of Testing of Hypothesis

Day and Date : Thursday, 23-4-2015 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q.No. (**1**) Q. No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**)*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select correct alternative

i) Type I error is defined as

A) reject H_0 when H_0 is false
B) reject H_0 when H_0 is true
C) both (A) and (B)
D) none of the above

ii) If the Test function $\phi(x) = \begin{cases} 1 & \text{if } x > c \\ 0 & \text{otherwise} \end{cases}$ then the test is

A) randomised B) non-randomised
C) both (A) and (B) D) neither (A) nor (B)

iii) Testing a simple hypothesis H_0 against a simple alternative H_1, let the power of the MP tests at level α and α' be β and β'. Then always

A) $\beta \geq \beta'$ B) $\beta \leq \beta'$ C) $\beta = \beta'$ D) $\beta \neq \beta'$

iv) For testing $H_0 : \theta \geq \theta_0$ vs $H_1 : \theta < \theta_0$ or $H_0 : \theta \leq \theta_0$ vs $H_1 : \theta > \theta_0$, the UMP test exists for the family of distribution

A) belongs one parameter exponential family
B) has an MLR property
C) either (A) or (B)
D) none of the above

v) Which statement is true ?

A) Every similar test has a Neyman-structure

B) Tests with Neyman-structure is a similar test

C) Both (A) and (B)

D) Neither (A) nor (B)

B) Fill in the blank :

i) The family of U(0, θ) distribution has MLR in ______________, when sample of size 'n' is available from U(0, θ).

ii) Likelihood ratio test for testing $H_0 : \theta \in \Theta_0$ vs $H_1 : \theta \in \Theta_1$ is defined as ___________

iii) MLR property of the distribution is used to obtain ____________ tests.

iv) If $\lambda(x)$ denotes the likelihood ratio statistic, then the asymptotic distribution of $-2 \log \lambda(x)$, under certain regularity conditions is _____________

v) UMP test leads to ____________ confidence intervals.

C) State whether the following statements are **true** or **false**.

i) UMP test always exist

ii) Test with Neyman-structure is a subset of similar test.

iii) There is no difference between level and size of a test.

iv) If ϕ is a test function then $(1-\phi)$ is also a test function. **(5+5+4)**

2. a) Answer the following.

i) Define simple and composite hypothesis. Give one example each.

ii) Explain shortest length confidence interval.

b) Write short notes on the following.

i) Chi-square test for contingency table

ii) Likelihood ratio test and MP test. **(6+8)**

3. a) State and prove the sufficiency part of Neyman-Pearson lemma.

b) Let x be a random sample with p.d.f.'s

$$f_0(x) = 1 \quad 0 \le x \le 1$$
$$= 0 \quad \text{otherwise}$$

and

$$f_1(x) = 4x \quad 0 \le x \le 1/2$$
$$= 4 - 4x \quad \frac{1}{2} \le x \le 1$$
$$= 0 \quad \text{otherwise}$$

on the basis of one observation, obtain the MP test of $H_0 : f = f_0$ against $H_1 : f = f_1$ at level $\alpha = 0.05$. What is the power of M.P. test ? **(7+7)**

4. a) Show that for p.d.f.'s $f_\theta(x)$ which have MLR property in $T(x)$, there exist an UMP test of size α for testing $H_0 : \theta \le \theta_0$ against $H_1 : \theta > \theta_0$.

b) Let $X_1, X_2 \ldots.. X_n$ be iid $N(\theta, \sigma^2)$ where σ^2 is known consider the testing problem $H_0 : \theta = \theta_0$ against $H_1 : \theta \ne \theta_0$. Show that a UMP size-$\alpha$ test for testing this problem does not exists. **(7+7)**

5. a) Define the terms.

I) Confidence coefficient of a confidence set

II) UMA confidence set

III) Unbiased confidence set

b) Obtain a UMA confidence interval for θ based on a random sample of size n from $U(0, \theta)$. **(6+8)**

6. a) Describe the Wilcoxon Signed -Rank test for single sample of size n.

   b) Define

      i) Similar test

      ii) Test with Neyman-structure

      iii) UMP α -similar test. Describe the method of obtaining similar test. **(6+8)**

7. a) Let X_1, X_2.... X_n be a random sample of size n from a normal population with mean μ and unknown variance σ^2. Derive LRT test to test $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$.

   b) Write short notes on the following.

      i) Generalised N-P lemma

      ii) MLR property. **(8+6)**

Seat No.	

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STATISTICS (Paper – IX) (Old)
Theory of Testing of Hypothesis

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*4) Figures to the **right** indicate **full** marks.*

1. A) Select correct alternative

i) Type I error is defined as
A) reject H_0 when H_0 is false
B) reject H_0 when H_0 is true
C) both (A) and (B)
D) none of the above

ii) If the Test function $\phi(x) = \begin{cases} 1 & \text{if } x > c \\ 0 & \text{otherwise} \end{cases}$ then the test is
A) randomised B) non-randomised
C) both (A) and (B) D) neither (A) nor (B)

iii) Testing a simple hypothesis H_0 against a simple alternative H_1, let the power of the MP tests at level α and α' be β and β'. Then always
A) $\beta \geq \beta'$ B) $\beta \leq \beta'$ C) $\beta = \beta'$ D) $\beta \neq \beta'$

iv) For testing $H_0 : \theta \geq \theta_0$ vs $H_1 : \theta < \theta_0$ or $H_0 : \theta \leq \theta_0$ vs $H_1 : \theta > \theta_0$, the UMP test exists for the family of distribution
A) belongs one parameter exponential family
B) has an MLR property
C) either (A) or (B)
D) none of the above

v) Which statement is true ?

A) Every similar test has a Neyman-structure

B) Tests with Neyman-structure is a similar test

C) Both (A) and (B)

D) Neither (A) nor (B)

B) Fill in the blank :

i) The family of U(0, θ) distribution has MLR in ______________, when sample of size 'n' is available from U(0, θ).

ii) Likelihood ratio test for testing $H_0 : \theta \in \Theta_0$ vs $H_1 : \theta \in \Theta_1$ is defined as ___________

iii) MLR property of the distribution is used to obtain ____________ tests.

iv) If $\lambda(x)$ denotes the likelihood ratio statistic, then the asymptotic distribution of $-2 \log \lambda(x)$, under certain regularity conditions is _____________

v) UMP test leads to ____________ confidence intervals.

C) State whether the following statements are **true** or **false**.

i) UMP test always exist

ii) Test with Neyman-structure is a subset of similar test.

iii) There is no difference between level and size of a test.

iv) If ϕ is a test function then $(1-\phi)$ is also a test function. **(5+5+4)**

2. a) Answer the following.

i) Define simple and composite hypothesis. Give one example each.

ii) Explain shortest length confidence interval.

b) Write short notes on the following.

i) Chi-square test for contingency table

ii) Likelihood ratio test and MP test. **(6+8)**

3. a) State and prove the sufficiency part of Neyman-Pearson lemma.

   b) Let x be a random sample with p.d.f.'s

$$f_0(x) = 1 \quad 0 \le x \le 1$$
$$= 0 \quad \text{otherwise}$$

and

$$f_1(x) = 4x \quad 0 \le x \le 1/2$$
$$= 4 - 4x \quad \frac{1}{2} \le x \le 1$$
$$= 0 \quad \text{otherwise}$$

on the basis of one observation, obtain the MP test of $H_0 : f = f_0$ against $H_1 : f = f_1$ at level $\alpha = 0.05$. What is the power of M.P. test ? **(7+7)**

4. a) Show that for p.d.f.'s $f_\theta(x)$ which have MLR property in $T(x)$, there exist an UMP test of size α for testing $H_0 : \theta \le \theta_0$ against $H_1 : \theta > \theta_0$.

   b) Let X_1, X_2 ..... X_n be iid $N(\theta, \sigma^2)$ where σ^2 is known consider the testing problem $H_0 : \theta = \theta_0$ against $H_1 : \theta \ne \theta_0$. Show that a UMP size-$\alpha$ test for testing this problem does not exists. **(7+7)**

5. a) Define the terms.

   I) Confidence coefficient of a confidence set

   II) UMA confidence set

   III) Unbiased confidence set

   b) Obtain a UMA confidence interval for θ based on a random sample of size n from $U(0, \theta)$. **(6+8)**

6. a) Describe the Wilcoxon Signed -Rank test for single sample of size n.

   b) Define

      i) Similar test

      ii) Test with Neyman-structure

      iii) UMP α -similar test. Describe the method of obtaining similar test. **(6+8)**

7. a) Let $X_1, X_2 .... X_n$ be a random sample of size n from a normal population with mean μ and unknown variance σ^2. Derive LRT test to test $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$.

   b) Write short notes on the following.

      i) Generalised N-P lemma

      ii) MLR property. **(8+6)**

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2014
STATISTICS (Paper – IX)
Theory of Testing of Hypotheses

Day and Date : Tuesday, 29-4-2014 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. (**1**) and Q. No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**).*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select correct alternative : **5**

1) Which one of the following is second kind error in testing of hypothesis ?
a) Accept H_0 b) Reject H_0
c) Accept H_0 when it is false d) Reject H_0 when it is true

2) For testing simple hypothesis against simple alternative, which of the followings values of α and β are not correct ?
a) $\alpha = 0.05, \beta = 0.70$ b) $\alpha = 0.05, \beta = 0.99$
c) $\alpha = 0.1, \beta = 0.73$ d) $\alpha = 0.5, \beta = 0.5$

3) For $N(\theta, 1)$, the pivot quantity is
a) $n\overline{X}$ b) $\sqrt{n}\,\overline{X}$ c) $n\left(\overline{X}-\theta\right)$ d) $\sqrt{n}\left(\overline{X}-\theta\right)$

4) The distribution $U\left(0, \theta\right)$
a) belong to one parameter exponential family
b) has an MLR property
c) both (a) and (b)
d) neither (a) nor (b)

5) A test function $\phi\left(x\right) \equiv \alpha$ for all x, has power
a) zero b) $1 - \alpha$ c) α d) one

1. B) Fill in the blanks : **5**

   1) The distribution $f(x, \theta) = \frac{e^{-(x-\theta)}}{\left[1 + e^{-(x-\theta)}\right]^2}$, $x \in R$ has MLR in

   2) If ϕ_1 and ϕ_2 are two tests of size α each then size of $\lambda\ \phi_1\ (x) + (1 - \lambda)\ \phi_2\ (x)$ is

   3) The acceptance region of UMPU size α test leads to __________ confidence set.

   4) If frequency of all classes is same then value of χ^2 is

   5) Neyman-Pearson lemma is used to construct ___________ tests.

1. C) State whether the following statements are **true** or **false** : **4**

   1) Type II error is more serious.

   2) MP test need not be unique.

   3) For testing simple against simple alternative LRT and MP test are different.

   4) Unbiased test rejects a true H_0 more often than false H_0.

2. a) Answer the following : **6**

   1) Define (i) MP test (ii) Unbiased test. Prove or disprove : MP test is unbiased.

   2) What is goodness of fit test ? Give its applications.

   b) Write short notes on the following : **8**

   i) Randomized and non-randomized tests.

   ii) Test for independence of attributes.

3. a) Define MLR property of family of distributions. Show that $U(0, \theta)$ possesses MLR property.

   b) Let $X_1, X_2, \ldots, X_n$ be iid Poisson (λ), $\lambda > 0$. Obtain MP test for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1\ (\lambda_1 > \lambda_0)$ with level α. Also find power of test.

   (7+7)

4. a) Define UMPU test. Prove that every UMP test is UMPU of same size.

   b) Obtain the UMPU level α test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$ based on $N(\theta, \sigma^2)$, where σ^2 is known for a sample of size n. **(6+8)**

5. a) Define shortest length confidence interval. Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(0, \theta)$ distribution. Obtain shortest length confidence interval for θ.

b) Let $X_1, X_2, \ldots, X_n$ be a sample from $N(\mu, \sigma^2)$ where σ^2 is known for testing $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$. Find UMA $(1-\alpha)$ level confidence sets for μ. **(8+6)**

6. a) Describe LRT procedure for testing $H_0 : \theta \in \Theta_0$ against $H_1 : \theta \in \Theta_1$. State large sample properties of LRT.

b) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N(\theta, \sigma^2)$, σ^2 is unknown. Derive LRT to test $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$. **(7+7)**

7. a) Describe Wilcoxon's signed rank test.

b) State generalized Neyman-Pearson lemma and give its applications. **(8+6)**

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – IX) (Old)
Theory of Testing of Hypothesis

Day and Date : Thursday, 23-4-2015 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt* ***five*** *questions.*
*2) Q.No. (**1**) Q. No. (**2**) are* ***compulsory****.*
3) Attempt ***any three*** *from Q. No. (**3**) to Q. No. (**7**)*
4) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Select correct alternative

i) Type I error is defined as
A) reject H_0 when H_0 is false
B) reject H_0 when H_0 is true
C) both (A) and (B)
D) none of the above

ii) If the Test function $\phi(x) = \begin{cases} 1 & \text{if } x > c \\ 0 & \text{otherwise} \end{cases}$ then the test is
A) randomised B) non-randomised
C) both (A) and (B) D) neither (A) nor (B)

iii) Testing a simple hypothesis H_0 against a simple alternative H_1, let the power of the MP tests at level α and α' be β and β'. Then always
A) $\beta \geq \beta'$ B) $\beta \leq \beta'$ C) $\beta = \beta'$ D) $\beta \neq \beta'$

iv) For testing $H_0 : \theta \geq \theta_0$ vs $H_1 : \theta < \theta_0$ or $H_0 : \theta \leq \theta_0$ vs $H_1 : \theta > \theta_0$, the UMP test exists for the family of distribution
A) belongs one parameter exponential family
B) has an MLR property
C) either (A) or (B)
D) none of the above

v) Which statement is true ?
   A) Every similar test has a Neyman-structure
   B) Tests with Neyman-structure is a similar test
   C) Both (A) and (B)
   D) Neither (A) nor (B)

B) Fill in the blank :

i) The family of U(0, θ) distribution has MLR in _______________, when sample of size 'n' is available from U(0, θ).

ii) Likelihood ratio test for testing $H_0 : \theta \in \Theta_0$ vs $H_1 : \theta \in \Theta_1$ is defined as ___________

iii) MLR property of the distribution is used to obtain _____________ tests.

iv) If $\lambda(x)$ denotes the likelihood ratio statistic, then the asymptotic distribution of $-2 \log \lambda(x)$, under certain regularity conditions is ______________

v) UMP test leads to _____________ confidence intervals.

C) State whether the following statements are **true** or **false**.

i) UMP test always exist

ii) Test with Neyman-structure is a subset of similar test.

iii) There is no difference between level and size of a test.

iv) If ϕ is a test function then $(1- \phi)$ is also a test function. **(5+5+4)**

2. a) Answer the following.

i) Define simple and composite hypothesis. Give one example each.

ii) Explain shortest length confidence interval.

b) Write short notes on the following.

i) Chi-square test for contingency table

ii) Likelihood ratio test and MP test. **(6+8)**

3. a) State and prove the sufficiency part of Neyman-Pearson lemma.

b) Let x be a random sample with p.d.f.'s

$f_0(x) = 1 \quad 0 \le x \le 1$

$= 0 \quad \text{otherwise}$

and

$f_1(x) = 4x \quad 0 \le x \le 1/2$

$= 4 - 4x \quad \frac{1}{2} \le x \le 1$

$= 0 \quad \text{otherwise}$

on the basis of one observation, obtain the MP test of $H_0 : f = f_0$ against $H_1 : f = f_1$ at level $\alpha = 0.05$. What is the power of M.P. test ? **(7+7)**

4. a) Show that for p.d.f.'s $f_\theta(x)$ which have MLR property in $T(x)$, there exist an UMP test of size α for testing $H_0 : \theta \le \theta_0$ against $H_1 : \theta > \theta_0$.

b) Let $X_1, X_2 ..... X_n$ be iid $N(\theta, \sigma^2)$ where σ^2 is known consider the testing problem $H_0 : \theta = \theta_0$ against $H_1 : \theta \ne \theta_0$. Show that a UMP size-$\alpha$ test for testing this problem does not exists. **(7+7)**

5. a) Define the terms.

I) Confidence coefficient of a confidence set

II) UMA confidence set

III) Unbiased confidence set

b) Obtain a UMA confidence interval for θ based on a random sample of size n from $U(0, \theta)$. **(6+8)**

6. a) Describe the Wilcoxon Signed -Rank test for single sample of size n.

   b) Define

      i) Similar test

      ii) Test with Neyman-structure

      iii) UMP α -similar test. Describe the method of obtaining similar test. **(6+8)**

7. a) Let $X_1, X_2 .... X_n$ be a random sample of size n from a normal population with mean μ and unknown variance σ^2. Derive LRT test to test $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$.

   b) Write short notes on the following.

      i) Generalised N-P lemma

      ii) MLR property. **(8+6)**

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Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2016
STATISTICS (Paper – IX)
Theory of Testing of Hypotheses (New CBCS)

Day and Date : Wednesday, 6-4-2016 Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative :

1) The hypothesis under test is

a) simple hypothesis b) alternative hypothesis
c) null hypothesis d) none of the above

2) Neyman Pearson Lemma provides

a) Most powerful test b) Chi-square test
c) Biased test d) None of these

3) If in Wilcoxon's signed rank test, the sample size is large, the statistic T^+ is distributed with mean

a) $\frac{n(n+1)}{4}$ b) $\frac{n(n+1)}{2}$
c) $\frac{n(2n+1)}{4}$ d) $\frac{n(n-1)}{4}$

4) A non-randomized test function takes values

a) – 1 or + 1 b) – 1 or 0
c) 0 or 1 d) none of these

5) Let $\lambda(x)$ denote the likelihood ratio statistic then asymptotic distribution of $-2 \log \lambda(x)$, under certain regularity condition.

a) Uniform b) Exponential
c) Normal d) Chi-square

B) Fill in the blanks :

1) If $X_1, X_2 \ldots X_n$ are iid exponential r.v.'s unknown mean θ . Then this family has MLR property in ____________

2) Type I error is rejecting the hypothesis H_0 when it is _________

3) Completeness family of distribution implies ____________completeness.

4) The degree of freedom associated with a 6×5 contingency table is ________

5) An MP test has power _________ than level.

C) State whether the following statements are **true** or **false** :

1) UMAU stands for uniformly most approximate unbiased.

2) One parameter exponential family does not possess MLR property.

3) UMP test always exist.

4) If ϕ is a test function then ϕ^2 is also a test function. **(5+5+4)**

2. a) Explain the following terms :

1) Size and power of a test

2) U-statistic.

b) Write short notes on the following :

a) Sign test

b) Generalized Neyman-Pearson Lemma. **(6+8)**

3. a) Define most powerful (M.P.) test. Illustrate with an example, M.P. test is not unique.

b) Construct M.P. test size α for testing $H_0 : \theta = 1$ Vs $H_1 : \theta = 0$ based on single observation from

$f_\theta(x) = 2x\ \theta + 1 - \theta)\ 0 < x < 1.$

Also, find power of a test. **(7+7)**

4. a) Define monotone likelihood ratio property. Check whether $U(0, \theta)$ has this MLR property.

b) Let $X_1, X_2 \ldots X_n$ be iid $N(\theta, 1)$. Obtain UMP test size α for testing $H_0 : \theta \leq \theta_0$ Vs $H_1 : \theta > \theta_0$. **(7+7)**

5. a) Define UMPU test. Develop UMPU test for $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \neq \sigma_0^2$ based on a r. s. of size n taken from $N(0, \sigma^2)$.

   b) Define similar test and Neyman structure test. Prove that a test with Neyman structure is similar. **(7+7)**

6. a) Explain :

   I) Confidence set

   II) Confidence coefficient

   III) UMA family of confidence set.

   b) Let $X_1, X_2 \ldots X_n$ be a sample from $N(\mu, \sigma^2)$, σ^2 is known for testing $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$. Find a UMA ($1 - \alpha$) level confidence set for μ . **(7+7)**

7. a) Let $X_1, X_2 \ldots X_n$ be r.s. of size n from $N(\mu, \sigma^2)$. Derive LRT of $H_0 : \mu \geq \mu_0$ against $H_1 : \mu < \mu_0$ when σ^2 known.

   b) Write a brief note on ‘goodness of fit problem’. **(7+7)**

SLR-MM – 519

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – IX) (CGPA) (Old)
Theory of Testing of Hypotheses

Day and Date : Tuesday, 24-11-2015 Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

***Instructions** : 1) Attempt **five** questions.*
*2) Q.No. (**1**) and Q.No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q.No. (**3**) to Q.No. (**7**).*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative : **5**

1) Which of the following is simple hypothesis for $N(\theta, \sigma^2)$?

a) $H_0 : \theta = 10$
b) $H_0 : \theta = 0, \sigma > 1$
c) $H_0 : \theta = 5, \sigma = 2$
d) $H_0 : \theta \neq 3, \sigma = 1$

2) If α and β are probabilities of type I and type II errors respectively. Which of the following inequality is satisfied by MP test ?

a) $\alpha < \beta$ b) $\alpha > \beta$ c) $\alpha + \beta > 1$ d) $\alpha + \beta \leq 1$

3) Degrees of freedom for a χ^2 in case of contingency table of order (4×3) are

a) 3 b) 6 c) 9 d) 12

4) The p.d.f. $f(x) = \frac{1}{2} e^{-|x-\theta|}, -\infty < x < \infty$ has MLR in

a) x b) $-x$ c) $|x|$ d) x^2

5) For LRT, asymptotic distribution of $-2\log\lambda$ is

a) normal b) t c) F d) χ^2

P.T.O.

B) Fill in the blanks. **5**

1) A good confidence set should have ________ length.
2) Based on single observation x from logistic distribution has MLR in ________
3) Probability of rejecting H_0 when it is false is called ________ of test.
4) Let $X \sim U(0, \theta)$. Then $H : \theta \leq 5$ is __________ hypothesis.
5) Acceptance region of _________ test leads to UMA confidence set.

C) State whether the following statements are **true** or **false**. **4**

1) Cauchy (1, θ) posses MLR property.
2) LRT is UMPU test.
3) A class of α-similar tests is a subclass of all unbiased size α tests.
4) If ϕ is randomized test then $(1 - \phi)$ is also randomized test.

2. a) Answer the following : **6**

1) Define :
   i) Size of test
   ii) Power of test
2) Explain likelihood ratio test procedure.

b) Write short notes on the following : **8**

i) Sign test
ii) Unbiased test.

3. a) State Neyman-Pearson Lemma and prove sufficient condition for a test to be most powerful.

b) Let X be a discrete random variable having two possible p.m.f.s given by

X	:	0	1	2	3	4
$P_0(x)$	:	0.2	0.3	0.1	0.1	0.3
$P_1(x)$	:	0.1	0.2	0.2	0.2	0.3

Obtain MP test of size $\alpha = 0.05$ for testing $H_0 : X \sim P_0(x)$ against $H_1 : X \sim P_1(x)$ on the basis of random sample of size one. Also compute power of test. **(7+7)**

4. a) Define MLR property of a family of distributions. Explain the use of MLR in construction of UMP test with the help of suitable example.

   b) Let $X_1, X_2, ..., X_n$ be a random sample of size n from $N(\theta, 1)$. Obtain UMP level α test for $H_0 : \theta \leq \theta_0$ against $H_1 : \theta \leq \theta_0$. **(7+7)**

5. a) Define UMPU test. Prove that every UMP test is UMPU of same size.

   b) Let $X_1, X_2, ... X_n$ be iid $U(0, \theta)$. Consider the following test for the $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$.

$$\phi(x) = \begin{cases} 1, \text{ if } \ x_{(n)} > \theta_0 \text{ or } x_{(n)} < \theta_0 \, \alpha^{\frac{1}{n}} \\ 0, \text{ otherwise} \end{cases}$$

   Examine whether ϕ is UMP. **(6+8)**

6. a) Define UMA confidence interval. Obtain one sided confidence interval for θ based on n independent observations from exponential distribution with mean θ.

   b) Derive LRT test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$ based on random sample of size n from $N(\theta, 1)$ distribution. **(7+7)**

7. a) Describe the test for independence of attributes.

   b) Describe Wilcoxon's signed-rank test. **(7+7)**

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