SLR-JS-367

Seat No.	

Set P

M.Sc. (Semester – I) (CBCS) Examination Oct/Nov-2019
Statistics
REAL ANALYSIS

Day & Date: Monday, 18-11-2019 Max. Marks: 70
Time: 11:30 AM To 02:00 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. **14**

1) The closed set includes all of its _____ points.
a) interior b) limit
c) member d) none of these

2) If A and B are open sets, then A ∪ B is _____.
a) always open b) always closed
c) may or not be open d) neither open nor closed

3) A set is said to be closed, if _____.
a) it includes all of its interior points
b) if every point of set is its limit point
c) if it includes all of its limit points
d) none of these

4) A compact set is always _____.
a) bounded b) closed
c) both (a) and (b) d) none of these

5) A convergence limit for a sequence is _____.
a) necessarily unique b) not necessarily unique
c) both (a) and (b) d) none of these

6) If a set is open, then its compliment _____.
a) has to be open b) may or may not be open
c) has to be closed d) all of these

7) The set of natural numbers is _____.
a) bounded above b) bounded below
c) both (a) and (b) d) bounded

8) The finite union of finite sets is _____.
a) finite b) countably infinite
c) uncountable d) may be finite or countable

9) A point c is said to be extremum point of function f, if _____.
a) $f^{'}(c) = 0$ b) $f(c) = 0$
c) $f^{'}(c) \neq 0$ d) none of these

10) The sequence $S_n = \sin\left(\frac{2\pi}{n}\right), n \in N$ is = _____.
a) convergent to 1 b) oscillatory
c) convergent to 0 d) none of these

11) The function $f(x) = 2 - x + x^2$ has extrema at the point _____.
a) $\frac{1}{2}$
b) 1
c) $\frac{1}{37}$
d) None of these

12) A continuous function is _____.
a) always differentiable
b) always right continuous
c) always bounded
d) all of these

13) If A is finite set and A U B is countable set, then _____.
a) B must be countable
b) B may or may not be countable
c) B is finite
d) none of these

14) A geometric series with common ratio r converges, if _____.
a) $|r| > 1$
b) $|r| < 1$
c) $r = 1$
d) all of these

Q.2 A) Answer the following questions. (Any Four) **08**
1) Define and illustrate countable set.
2) Define and illustrate convergent sequence.
3) Define and illustrate compact set.
4) State and prove necessary condition for convergence of a series.
5) Define and illustrate concept of limit point.

B) Write notes. (Any Two) **06**
1) Cauchy sequence
2) Mean value theorem
3) Geometric series/

Q.3 A) Answer the following questions. (Any Two) **08**
1) Check whether following series are convergent.
i) $\sum_{n=1}^{\infty} \frac{x^n}{n!}$
ii) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$
2) Explain any two tests for convergence of a series.
3) Prove that the set $[0,1]$ is uncountable.

B) Answer the following questions. (Any One) **06**
1) Explain how to calculate Riemann integration of a continuous function.
2) Prove: Countable union of countable sets is countable.

Q.4 A) Answer the following questions. (Any Two) **10**
1) Explain Lagrange's method for obtaining constrained maxima or minima.
2) State and prove fundamental theorem on calculus.
3) Prove that a set is closed, if and only if its compliment is open.

B) Answer the following questions. (Any One) **04**
1) State Taylor's theorem. Find the power series expansion for the following functions:
a) $f(x) = e^x$
b) $f(x) = e^{-x}$
2) Define radius of convergence. Also find it for the following power series.

$$1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \cdots$$

Q.5 Answer the following questions. (Any Two) **14**

a) Find the stationary value of $x^2 + y^2 + z^2$ subject to condition $x^3 + y^3 + z^3 = 3a^3$.

b) Find upper Riemann integral and lower Riemann integral of $f(x) = x^2$ over 1 to 2 and conclude whether the function is Riemann integrable.

c) Explain limit superior and limit inferior of a sequence. Also give illustration.

SLR-MM – 502

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2015
STATISTICS (Paper – II)
Real Analysis (New CBCS)

Day and Date : Wednesday, 18-11-2015 Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions *: 1) Attempt **five** questions.*
*2) Q. No. (**1**) and Q. No. (**2**) are **Compulsory**.*
*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**).*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) The finite intersection of open sets is __________
a) An open set b) A closed set
c) Both open and closed d) Neither open nor closed

2) Subset of a countable set is __________
a) Always countable b) Always uncountable
c) May or may not be countable d) None of these

3) The collection of all the limit points of a set is called its ___________
a) Interior set b) Derival set
c) Neighbourhood d) None of these

4) The function $f(x) = |x|$ is __________
a) Step function b) Continuous
c) Discontinuous at zero d) None of these

5) Every Cauchy sequence is a ___________ sequence.
a) Divergent b) Convergent
c) Monotonic d) Oscillatory

P.T.O.

B) Fill in the blanks : **5**

1) A set is closed if it includes all of its __________ points.

2) Least upper bound of a set is also called as __________

3) For an open set, every point of the set is its ___________ point.

4) Countable union of countable sets is ___________

5) Finite union of closed sets is always ___________

C) State whether the following statements are **True** or **False** : **4**

1) Root test can be applied for any series to check its convergence.

2) If exists, infimum is always unique.

3) Arbitrary union of closed sets is always closed.

4) Set of integers is a countable set.

2. a) State the following :

i) Cauchy criterion of convergence of a series.

ii) Bolzano-Weistrauss theorem

iii) Heine-Borel theorem.

b) Write short note on the following :

i) Mean value theorem.

ii) Limit superior of a sequence. **(6+8)**

3. a) Prove that a set is open iff its compliment is closed.

b) Prove or disprove : Arbitrary union of open sets is open.

c) Show that the set of rationals is a countable set. **(5+5+4)**

4. a) Prove or disprove : Monotonic bounded sequence always converges.

b) Examine the convergence of following sequences :

i) $S_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \dots \frac{1}{n!}$ for all $n \in N$

ii) $S_n = n^{1/n}$ for all $n \in N$. **(8+6)**

5. a) Describe comparison test and ratio test of convergence of a series.

   b) Describe Lagrange's method of undetermined multipliers. **(7+7)**

6. a) Define lower and upper Riemann integral of a function f(x). Also state the condition under which function is said to be Riemann integrable.

   b) Check whether following functions are Riemann integrable over (0, 1). If so find the integral.

      i) f(x) = 2x

      ii) f(x) = 2, if x is rational

      = 1, if x is irrational. **(7+7)**

7. a) Find liminf and limsup of the sequence $S_n = 1 + \frac{(-1)^n}{n}$. Hence discuss its convergence.

   b) Explain the term radius of convergence of a power series.

   c) State Lebnitz rule and its one application. **(8+3+3)**

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Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2015
STATISTICS (Paper – II)
Real Analysis (New)

Day and Date : Friday, 17-4-2015 Total Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative. **5**

1) A set may have
   a) No limit point
   b) A unique limit point
   c) Finite or infinite number of limit points
   d) All the above

2) The limit points of $S_n = 1 + (-1)^n$ are
   a) 1, 0 b) 0, 2
   c) 1, 1 d) 2, 1

3) The function $f(x) = x^2$ is
   a) Continuous b) Discontinuous
   c) Uniformly continuous d) None of these

4) The improper integral $\int_{-\infty}^{\infty} e^x dx =$
   a) 0 b) 1
   c) π d) ∞

5) The function f is bounded and integrable on [a, b] then f is
   a) Continuous on [a, b] b) Differentiable on [a, b]
   c) Both a) and b) d) Neither a) nor b)

B) Fill in the blanks : **5**

1) A set of all limit points of a set is called ________ set.
2) A set is closed if and only if its complement is _________
3) Every convergent bounded sequence has _______ limit.
4) If a power series converges for all values of x, then it is called _________ convergent.
5) The radius of convergence of series $1 + 2x + 3x^2 + 4x^3 + ...$is _________

C) State whether the following statements are **true** or **false** : **4**

1) The limit point of a set is always a member of that set.
2) A sequence cannot converge to more than one limit points.
3) Every power series is convergent for x = 0.
4) The function $f(x) = \frac{1}{2}$ is uniformly convergent on (0, 1].

2. a) State the following : **6**

i) Taylor's theorem
ii) Heine-Borel theorem
iii) Bolzano-Weierstrass theorem.

b) Write short notes on the following : **8**

i) Countable and uncountable sets.
ii) Radius of convergence.

3. a) Define open set. Give an example of an open set and other one which is not open set with justifications.

b) Prove that finite intersection of open sets is an open set.

c) Show that the set of real numbers in [0, 1] is uncountable. **(5+5+4)**

4. a) Define Cauchy sequence. Prove that every Cauchy sequence is convergent.

b) Examine the convergence of following sequence.

i) $S_n = \frac{1}{1!} + \frac{1}{2!} + ... + \frac{1}{n!}, \forall n \in N$

ii) $S_n = \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{n+n}, \forall n \in N$ **(6+8)**

5. a) Describe any four tests for convergence of series.

   b) Show that the series $x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots$ converges absolutely for all values of x.

   c) Show that for any fixed value of x, $\sum_{n=1}^{\infty}\frac{\sin nx}{n^2}$ is convergent. **(8+3+3)**

6. a) Define Riemann integral. Prove that every continuous function is integrable.

   b) Find the radius of convergence of the following series.

      i) $1-\frac{x}{2}+\frac{x^2}{3}-\frac{x^3}{4}+\ldots$

      ii) $x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots$ **(8+6)**

7. a) Find the minimum value of $x^2 + y^2 + z^2$ when $x + y + z = 3a$.

   b) Show that the function $f(x) = x^2$ is uniformly continuous on [–1, 1].

   c) Test the convergence of $\int_0^1 \frac{dx}{\sqrt{1-x^3}}$. **(6+4+4)**

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SLR-ES-365

Seat No.	

Set P

M.Sc. (Semester - I) (CBCS) Examination March/April-2019
Statistics
REAL ANALYSIS

Day & Date: Friday, 26-04-2019 Max. Marks: 70
Time: 12:00 PM To 02:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Choose Correct Alternative from the following. 14

1) Every subset of countable set is ______
a) countable b) uncountable
c) may or may not be countable d) finite

2) If A and B are closed sets, then A ∩ B is ______.
a) always closed b) always open
c) may or may not be open d) neither open nor closed

3) For a finite set with n elements, derived set contains _____ points.
a) zero b) exactly one
c) 2^n d) None of these

4) Every monotonic bounded sequence ______.
a) converges b) diverges
c) converges depending on bounds d) none of these

5) A subset of uncountable set _______.
a) is always uncountable b) is always countable
c) may or may not be countable d) none of these

6) Finite union of countable sets is ______.
a) always countable b) may or may not be countable
c) always uncountable d) none of these

7) If a set is open, then its compliment ______.
a) has to be open b) may or may not be open
c) has to be closed d) all of these

8) The set of integers is ______.
a) bounded b) countable
c) both (a) and (b) d) uncountable

9) A point c is said to be extremum point of function f, if ______.
a) $f'(c) = 0$ b) $f(c) = 0$
c) $f'(c) \neq 0$ d) None of these

10) The sequence $S_n = \sin(2n\pi), n \in N$ is ______.
a) convergent to 1 b) oscillatory
c) convergent to 0 d) none of these

11) The function $f(x) = -x^2 + 2x + 3$ has ______.
a) minimum at point $x = 1$ b) maximum at point $x = 1$
c) convergent to 0 d) none of these

12) A differentiable function is ______.
a) always continuous b) may or may not be continuous
c) always unbounded d) all of these

13) Which of the following is not a test for checking convergence of a series?
   a) Root test
   b) Ratio test
   c) Comparison test
   d) Cantor test

14) A geometric series with common ratio r converges, if ______.
   a) $|r| > 1$
   b) $|r| < 1$
   c) $r = 1$
   d) all of these

Q.2 A) Answer the following (Any Four) **08**

1) Define and illustrate infimum of a set.
2) Define and illustrate countable set.
3) Define and illustrate bounded sequence.
4) State
   i) Bolzano-Weistrauss theorem
   ii) Heine-Borel theorem
5) Define and illustrate concept of interior point.

B) Write Notes on (Any two) **06**

1) Taylor's theorem
2) Mean value theorem
3) Cauchy criterion for convergence of a sequence

Q.3 A) Answer the following (Any two) **08**

1) What is meant by convergent sequence? Prove that every monotonic non-increasing bounded below sequence is convergent.
2) Find limit inferior and limit superior of the sequence $\{S_n\}$,where
$$S_n = 2 + \frac{(-1)^n}{n}, n \in N$$
3) Define geometric series and verify its convergence for different values of common ratio.

B) Answer the following (Any one) **06**

1) Prove: Countable union of countable sets is always countable
2) Prove: Every convergent sequence is Cauchy sequence

Q.4 A) Answer the following (Any two) **10**

1) State and prove rule of integration by parts.
2) Explain Lagrange's method for obtaining constrained maxima or minima
3) What is meant by closed set? Prove that arbitrary intersection of closed sets is closed.

B) Answer the following (Any one) **04**

1) Define radius of convergence. Illustrate it using any power series.
2) State Taylor's theorem. Find the power series expansion for the following functions.
   i) $f(x) = e^x$
   ii) $f(x) = \sin x$

Q.5 Answer the following (Any two) **14**

a) Explain the concept of Riemann integration.

b) State and prove necessary condition for convergence of a series. Hence, or otherwise check whether following series is convergent.
$$\sum a_n = \sum_n \left(1 + \frac{1}{n}\right)^n$$

c) Examine the convergence of p-series for various values of p

SLR-MB – 604

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2016
STATISTICS (New CBCS)
Real Analysis (Paper – II)

Day and Date : Thursday, 31-3-2016 Total Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) The set of limit points for the set (–1, 2) is
a) (–1, 2) b) (0, 2) c) [0, 2] d) [–1, 2]

2) A closed set includes all of its
a) Interior points b) Limit points
c) Member points d) None of these

3) The function f(x) = |x| is
a) Continuous b) Discontinuous at zero
c) Step function d) None of these

4) Subset of a countable set is
a) Always countable
b) Always uncountable
c) May or may not be countable
d) None of these

5) A monotonic bounded sequence is always
a) Convergent b) Divergent
c) Oscillatory d) May or may not be convergent

P.T.O.

B) Fill in the blanks : **5**

1) A set is closed if and only if its compliment is __________
2) Finite union of open sets is ________
3) The set of all interior points of a set is called _________
4) The set of all limit points of a set is called ________
5) Finite union of countable sets is _________

C) State whether the following statements are **true** or **false** : **4**

1) Every point of a set is its interior point.
2) If exists, supremum is always unique.
3) Every monotonic sequence in R converges.
4) Every set has atleast one limit point.

2. a) State the following :

i) Cauchy criterion of convergence of a series.
ii) Bolzano-Weistrauss theorem.
iii) Lebnitz rule.

b) Write short note on the following :

i) Bounded set and infimum of a set.
ii) Limit inferior of a sequence. **(6+8)**

3. a) Define closed set. Is arbitrary intersection of closed sets always closed ? Justify.

b) Define countable set. Prove that countable union of countable sets is countable.

c) Show that the set of rationals is a countable set. **(5+5+4)**

4. a) Prove that a sequence is convergent, iff it is a Cauchy sequence.

b) Examine the convergence of following sequences :

i) $S_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots \frac{1}{n!}$ for all $n \in N$.

ii) $S_n = n^{1/n}$ for all $n \in N$. **(8+6)**

5. a) Describe any four tests of convergence of a series.

   b) Prove that the series $1/n^p$ diverges for $p \leq 1$ and converges for $p>1$. **(8+6)**

6. a) Define Riemann integral. Prove that every continuous function is integrable.

   b) Check whether following functions are Riemann integrable over (0, 1). If so, find the integral.

      i) $f(x) = |x|$

      ii) $f(x) = 1$, if x is rational
          $= 0$, if x is irrational. **(7+7)**

7. a) Find the minimum value of $x^2 + 2y^2 + 3z^2$ when $x + y + z = k$.

   b) Find liminf of the sequence $S_n = 1 + [(-1)^n/n]$, $n \in N$.

   c) Find limsup of the sequence $S_n = 1 - [(-1)^n/n]$, $n \in N$. **(8+3+3)**

SLR-BP – 468

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2015
STATISTICS (Paper – II)
Real Analysis (New)

Day and Date : Friday, 17-4-2015 Total Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative. **5**

1) A set may have
   a) No limit point
   b) A unique limit point
   c) Finite or infinite number of limit points
   d) All the above

2) The limit points of $S_n = 1 + (-1)^n$ are
   a) 1, 0 b) 0, 2
   c) 1, 1 d) 2, 1

3) The function $f(x) = x^2$ is
   a) Continuous b) Discontinuous
   c) Uniformly continuous d) None of these

4) The improper integral $\int_{-\infty}^{\infty} e^x dx =$
   a) 0 b) 1
   c) π d) ∞

5) The function f is bounded and integrable on [a, b] then f is
   a) Continuous on [a, b] b) Differentiable on [a, b]
   c) Both a) and b) d) Neither a) nor b)

P.T.O.

B) Fill in the blanks : **5**

1) A set of all limit points of a set is called ________ set.
2) A set is closed if and only if its complement is _________
3) Every convergent bounded sequence has _______ limit.
4) If a power series converges for all values of x, then it is called _________ convergent.
5) The radius of convergence of series $1 + 2x + 3x^2 + 4x^3 + ...$is _________

C) State whether the following statements are **true** or **false** : **4**

1) The limit point of a set is always a member of that set.
2) A sequence cannot converge to more than one limit points.
3) Every power series is convergent for x = 0.
4) The function $f(x) = \frac{1}{2}$ is uniformly convergent on (0, 1].

2. a) State the following : **6**

i) Taylor's theorem
ii) Heine-Borel theorem
iii) Bolzano-Weierstrass theorem.

b) Write short notes on the following : **8**

i) Countable and uncountable sets.
ii) Radius of convergence.

3. a) Define open set. Give an example of an open set and other one which is not open set with justifications.

b) Prove that finite intersection of open sets is an open set.

c) Show that the set of real numbers in [0, 1] is uncountable. **(5+5+4)**

4. a) Define Cauchy sequence. Prove that every Cauchy sequence is convergent.

b) Examine the convergence of following sequence.

i) $S_n = \frac{1}{1!} + \frac{1}{2!} + ... + \frac{1}{n!}, \forall n \in N$

ii) $S_n = \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{n+n}, \forall n \in N$ **(6+8)**

5. a) Describe any four tests for convergence of series.

   b) Show that the series $x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ converges absolutely for all values of x.

   c) Show that for any fixed value of x, $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$ is convergent. **(8+3+3)**

6. a) Define Riemann integral. Prove that every continuous function is integrable.

   b) Find the radius of convergence of the following series.

   i) $1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \ldots$

   ii) $x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ **(8+6)**

7. a) Find the minimum value of $x^2 + y^2 + z^2$ when $x + y + z = 3a$.

   b) Show that the function $f(x) = x^2$ is uniformly continuous on [–1, 1].

   c) Test the convergence of $\int_0^1 \frac{dx}{\sqrt{1 - x^3}}$. **(6+4+4)**

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SLR-VB – 2

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2014
STATISTICS (Paper – II)
Real Analysis

Day and Date : Wednesday, 23-4-2014 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative : **5**

i) Let A and B be countable sets. Then A – B

a) must be finite b) must be countable
c) must be empty d) none of the above

ii) The set of all rational numbers on the real line is

a) countable b) uncountable
c) finite d) none of the above

iii) A sequences of positive numbers unbounded above

a) necessarily converges b) necessarily diverges
c) may or may not converge d) none of the above

iv) The series $\sum_{n=1}^{\infty} \frac{1}{n^{1+\alpha}}$ converges for

a) $\alpha \leq 0$ b) $\alpha < 0$
c) $\alpha > 0$ d) $\alpha \geq 0$

v) The value of the integral $\int_0^1 x^2 dx^2$ is

a) $\frac{1}{2}$ b) $1/3$
c) 0 d) 2

P.T.O.

1. B) Fill in the blanks : **5**

   i) The product of any two uniformly continuous functions on set A is _________

   ii) The limit inferior of the sequence $\left(1+\frac{1}{n}\right)$ is __________

   iii) The series is $\sum \frac{1}{n(n+1)}$ is __________

   iv) The value of $\int_{0.6}^{3.2} d[x]$ is __________

   v) The set of limit points of the set (0, 1] is __________

1. C) State whether the statements are **true** or **false** : **4**

   i) Every Cauchy sequence converges.

   ii) The radius of convergence of the series $1 + x + x^2 + x^3 + ...$ is 2.

   iii) The set [0, 1) is compact.

   iv) If x is a limit point of A and $A \subset B$ then x is also a limit point of B.

2. A) i) Discuss the convergence of the sequence $\left\{\sqrt{2}-1, \sqrt{3}-\sqrt{2}, \sqrt{4}-\sqrt{3}, ...\right\}$.

   ii) Explain with suitable examples, the following terms :

   a) Neighbourhood of a point.

   b) Closure of a set. **(3+3=6)**

   B) Write short notes on the following :

   a) Vector and matrix differentiation.

   b) Integration by parts. **(4+4=8)**

3. A) Define Cauchy sequence verify whether the following sequences are Cauchy or not.

   a) $S_n = \frac{1}{n}$, n = 1, 2, ...

   b) $S_n = 1 + 2 + ... + n$, n = 1, 2,...

   B) Define open set. Prove that a set is open iff its complement in R is closed. **(7+7)**

4. A) Discuss the convergence of sequence $S_n = \left(1+\frac{1}{n}\right)^n$.

   B) Test the convergence of

   a) $\sum_{n=1}^{\infty} \frac{1}{n\log\left(1+\frac{1}{n}\right)}$

   b) $\frac{1}{2}+\frac{1}{2.2^2}+\frac{1}{3.2^3}+\frac{1}{4.2^4}+....$ **(7+7)**

5. A) Define Riemann-Steiltje's integral. Obtain $\int_0^1 x^2 d\alpha(x)$, where,

$$\alpha(x) = \begin{cases} x/2 & \text{if} \quad 0 \le x < 0.4 \\ 0.5 & \text{if} \quad 0.4 \le x < 0.6 \\ x & \text{if} \quad 0.6 \le x \le 1 \end{cases}.$$

   B) Explain the Lagrange's method of undetermined multipliers. Hence, minimize $x^2 + y^2 + z^2$ subject to constraint $x + y + z = 9$. **(7+7)**

6. A) Define uniform convergence. Prove that the sequence $f_n(x) = x^n$ converges uniformly on [0, 0.5].

   B) Discuss the convergence of the integral $\int_0^1 x^{m-1}(1-x)^{n-1}dx$. **(8+6)**

7. A) Evaluate the integral $\int_C dx\,dy\,dz$, where $C = \{(x, y, z) \mid 0 \le x, y, z \le 1, x + y + z \le 1\}$.

   B) Use the Taylor's series formula to expand i) log (1 + x). (ii) sin x. **(8+6)**
